Computing H-Partitions in ASP and Datalog

Chloé Capon, Nicolas Lecomte and Jef Wijsen

University of Mons, Belgium

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Let's investigate on a problem: finding H-Partitions of a graph.

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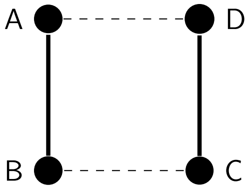
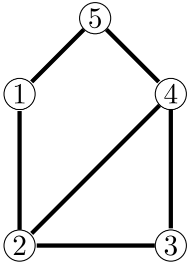
If there exists such a labeling, (H, G) is called a yes-instance; otherwise (H, G) is a no-instance.

H-Partitions

Example





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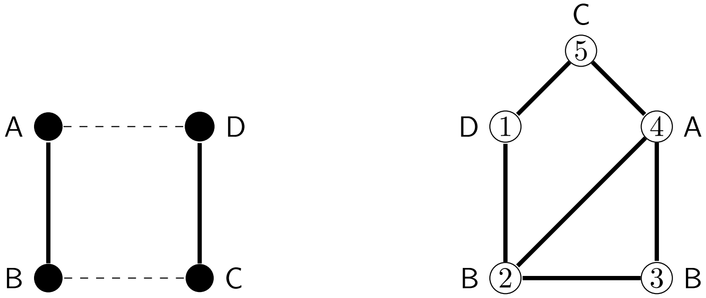
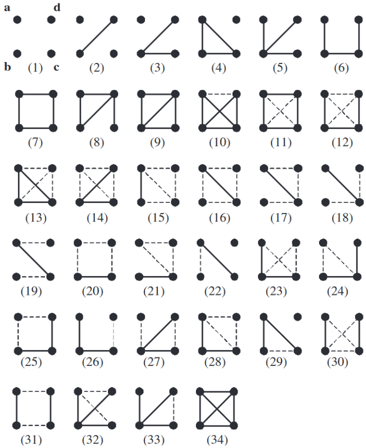
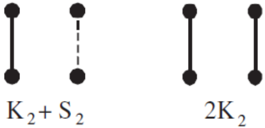


Model graphs

All possible model graphs (up to isomorphism):





Finding H-Partitions

- The H-partitioning problem for $K_2 + S_2$, is in polynomial time [dFKKR00, KR07];
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$\rightsquigarrow$ We experimentally compare a Datalog with stratified negation program with a guess-and-check ASP program.

ASP program

Guess and check

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% Constraints of the model graph.
:- placedIn(X,P), placedIn(Y,Q), full(P,Q), not e(X,Y).
:- placedIn(X,P), placedIn(Y,Q), dotted(P,Q), e(X,Y).
```

Datalog approach: Bases

Base

We say that the quadruplet (x_A, x_B, x_C, x_D) is a base for H if the subgraph of G induced by these vertices is a yes-instance of H-PARTITION(H) for a labeling where x_A, x_B, x_C, and x_D are respectively labeled by A, B, C, and D.

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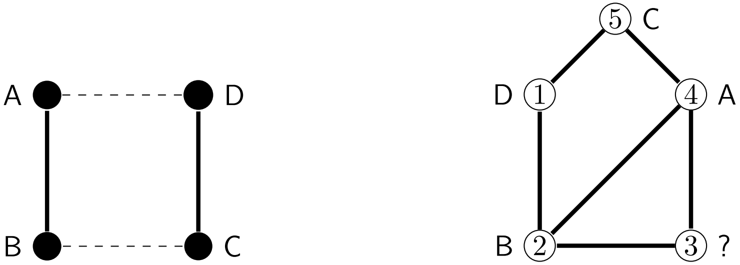

$(4, 2, 5, 1)$ is a base for H.

Datalog program

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For each base until we find a solution:

[1] for model graphs (7), (10) and (11) some additional tests are required.

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2. Check if this base can be extended to a complete labeling of all vertices; repeatedly pick an unlabeled vertex, and compute its possible labels:

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   - if only one label is possible $\implies$ label that vertex with it;
   - if no label is possible $\implies$ cannot be extended to a complete labeling.
3. If for every unlabeled vertex at least two labels remain possible then G is a yes-instance [1].

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Generating instances

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⇝ We need to distinguishing between yes-instances and no-instances.

Motivation:

- on a yes-instance, an algorithm can stop as soon as a labeling is found;
- on no-instances no such early stopping is possible.

Generation of no-instances

model graph	# yes	# no	model graph	# yes	# no
(1)	1000	0	(18)	999	1
(2)	1000	0	(19)	185	815
(3)	1000	0	(20)	15	985
(4)	998	2	(21)	35	965
(5)	15	985	(22)	1000	0
(6)	175	825	(23)	15	985
(7)	13	987	(24)	15	985
(8)	15	985	(25)	14	986
(9)	13	987	(26)	56	944
(10)	9	991	(27)	15	985
(11)	0	1000	(28)	5	995
(12)	0	1000	(29)	1000	0
(13)	0	1000	(30)	0	1000
(14)	2	998	(31)	0	1000
(15)	4	996	(32)	6	994
(16)	4	996	(33)	15	985
(17)	183	817	(34)	12	988

For most cases: repeatedly generating random graphs will quickly lead to a no-instance.

Generation of yes-instances

Let $a, b, c, d \in \mathbb{N}_0$ such that $a + b + c + d = n$. One can wonder if there exists a yes-instance with m edges where the numbers of vertices labeled by A, B, C, and D are, respectively, a, b, c, and d.

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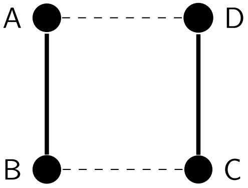
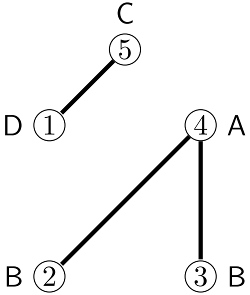
- G_{min} is the graph where the only edges are the ones following the full constraints from H;





Here, $n = 5$ with $a = c = d = 1$ and $b = 2$.

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- G_{min} is the graph where the only edges are the ones following the full constraints from H;
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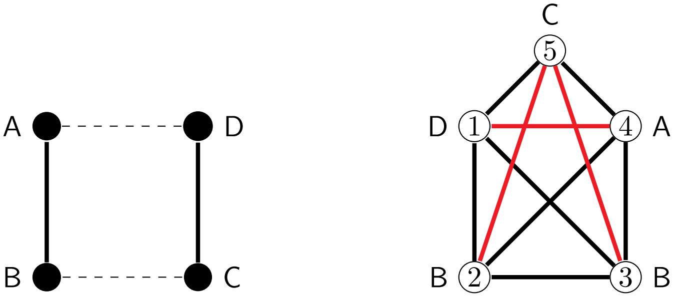

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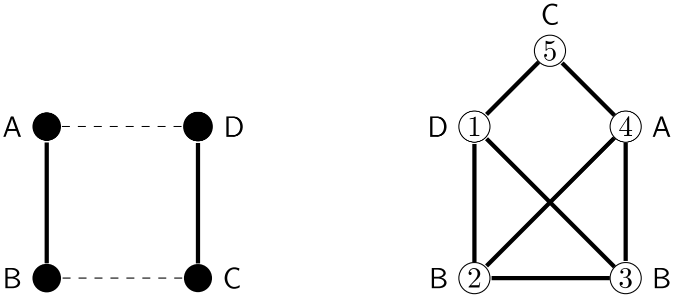

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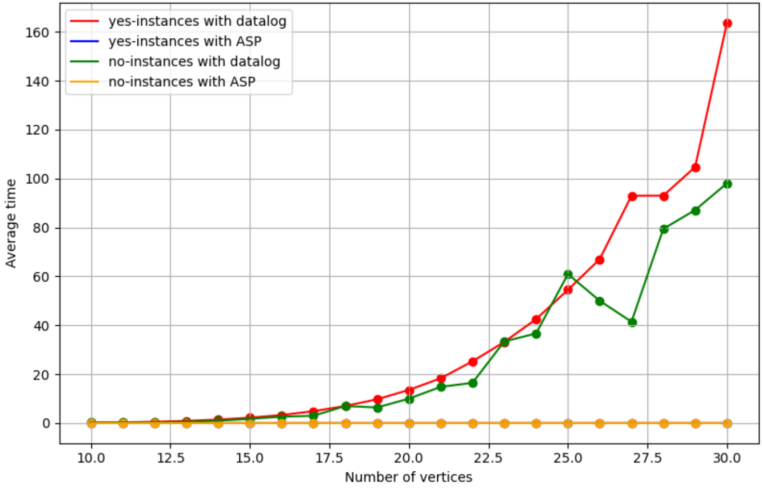
Theorem

Let H be a model graph. Let $a, b, c, d \in \mathbb{N}_0$ and $n = a + b + c + d$. Every graph G such that $G_{min} \subseteq G \subseteq G_{max}$ has a solution with a vertices labeled by A, b labeled by B, etc.

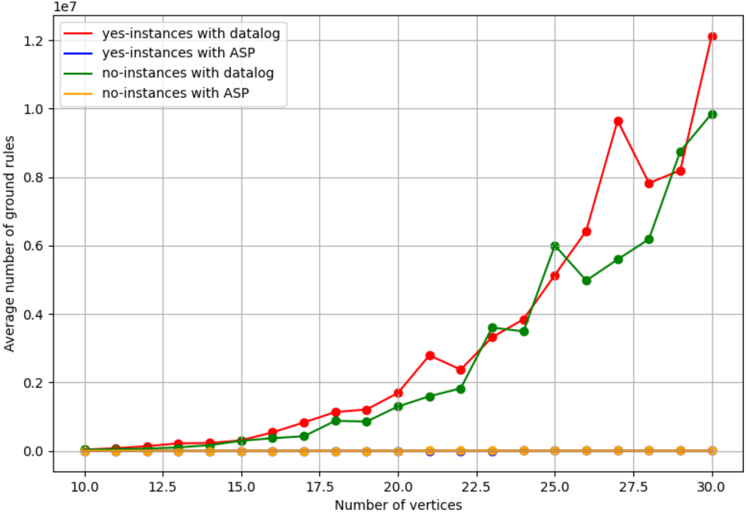
Experimentations

Time of resolution with Clingo



Experimentations

Number of ground rules with Clingo



Conclusion

- We have given two programs:
  1. a program in Datalog with stratified negation;
  2. a guess-and-check ASP program.
- We have generated yes and no-instances to compare the programs.

$\rightsquigarrow$ The Datalog approach is slower than the guess-and-check because it leads to too many ground rules.

Future work

- Automatic generation of no-instances;
- Test on a datalog engine.

Thank you for your attention!

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