Time Flies When Looking out of the Window: Timed Games with Window Parity Objectives

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Highlights 2021

Timed games

- We consider two-player games played on timed automata where the set of edges are partitioned between the players.
- At each step of the game, both players simultaneously present a delay (a non-negative real number) and an action and the play proceeds following a fastest move.

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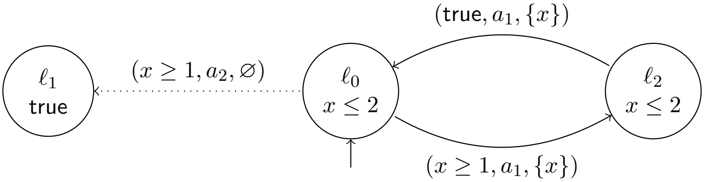


- Examples of start of plays:
  - $(\ell_0, 0)\,((1.5, a_1), (1, a_2))\,(\ell_1, 1)\ldots$
  - $(\ell_0, 0)\,((1, a_1), (1, a_2))\,(\ell_1, 1)\ldots$
  - $(\ell_0, 0)\,((1, a_1), (1, a_2))\,(\ell_2, 0)\ldots$

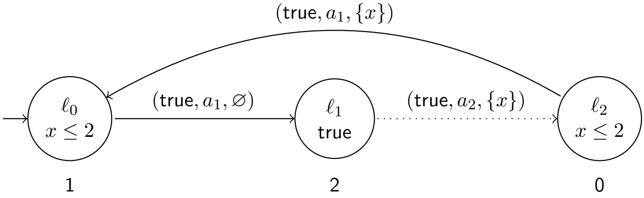
Objectives and winning

- Player 1 represents the system and wants to enforce a specification given as an objective, i.e., a set of good plays.
- In timed games, there can be plays where the overall sum of delays is bounded, called time-convergent plays. Plays were the overall sum of delays is unbounded are called time-divergent.
- We define winning conditions[1] as the set of plays such that one of the two following conditions is satisfied:
  - the play is time-convergent and player 1 is not responsible for the convergence;
  - the play is time-divergent and satisfies the objective.

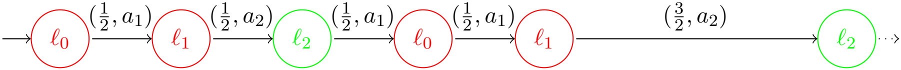
[1] de Alfaro et al., "The Element of Surprise in Timed Games", 2003 [dAFH+03].

Parity objectives

- In a timed parity game, each location of the timed automaton is labelled by a non-negative integer called a priority.
- The parity objective is the set of plays such that the smallest priority appearing infinitely often throughout the play is even.

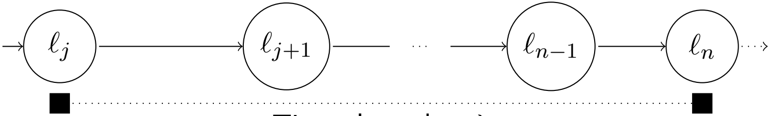


- There can be an arbitrarily large delay between odd priorities and smaller even priorities.



Windows to implement time constraints

- The window mechanism is a means of enforcing time constraints between odd priorities and smaller even priority.
- Window objectives are based on the notion of good windows.
- Fix a bound λ on the length of windows. A good window for the parity objective is a window in which:
  - strictly less than λ time units elapse and
  - the smallest priority appearing in the window is even.



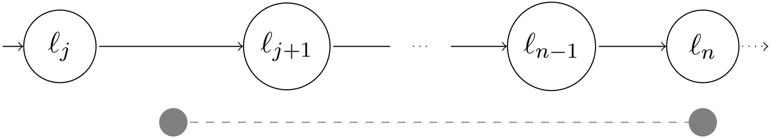
Time elapsed $< \lambda$

$\min_{j \leq k \leq n} p(\ell_k)$ is even

Timed window parity objectives

We have studied two timed window objectives.

- Direct timed window parity objective: there is a good window at all times. This objective requires good windows even in intermediate states occurring during delays.



- Timed window parity objective: the direct window parity holds from some point on.

Result overview

- We have an algorithm based on a reduction to a safety or co-Büchi timed game: the goal is to avoid locations indicating bad windows.
- The case of a conjunction of direct timed window parity objectives or a conjunction of timed window parity objectives can be handled with a similar approach.
- We have also considered these objectives in the context of verification of timed automata.

Complexity summary

	Single dimension	Multiple dimensions
Timed automata	PSPACE-complete	PSPACE-complete
Timed games	EXPTIME-complete	EXPTIME-complete

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History of window objectives

Window objectives have been studied in discrete-time settings:

- in turn-based games with mean-payoff and total-payoff objectives [CDRR15];
- in turn-based games with parity objectives [BHR16];
- in Markov decision processes for parity and mean-payoff objectives [BDOR20].

We extend window objectives to a continuous-time setting, for timed automata and timed games.

Overview of our work

- In a nutshell, the direct timed window parity objective requires, for a fixed bound λ on the size of windows, that at all times along a play, there is a window of size at most λ in which the smallest priority is even.
- We also consider a prefix-independent variant, requiring the direct objective to hold from some point forward.
- For these objectives, verification of timed automata is PSPACE-complete and realizability in timed games is EXPTIME-complete.