Sender-receiver stopping games

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Practical situation: A better informed advisor (sender) gives suggestions to a decision maker (receiver) when to take a decision (stopping the game). They interact repeatedly until the decision is taken.

Model

The game is given by

- 2 players: The sender and the receiver.
- Periods: $1, 2, 3, \ldots$.
- Set of states of the world: $I = [0, 1]$.
- Message space for the sender: $M = \{m_c, m_q\}$.
- Action space for the receiver: $A = \{a_c, a_q\}$.
- Strictly increasing and continuous functions f^t, g^t: $I \to \mathbb{R}_+$ at each period t. (These determine the payoffs if the receiver stops at period t.)

Timeline at period t

- State of the world θ^t is drawn uniformly from I.
- The sender -
  1. learns the state θ^t,
  2. sends a message $m^t \in M = \{m_c, m_q\}$ to the receiver.
- The receiver -
  1. sees the message from the sender,
  2. chooses an action $a^t \in A = \{a_c, a_q\}$.
- If the receiver
  1. takes action a_c: The game continues to period $t+1$.
  2. takes action a_q: The game stops at period t, the payoffs $f^t(\theta^t)$ and $g^t(\theta^t)$ are awarded to the sender and the receiver respectively.

Strategies

Markov strategies

- A strategy for the sender: $\sigma = (\sigma^t)_t$ where $\sigma^t : I \to \Delta(M)$ is measurable.
- A strategy for the receiver: $\tau = (\sigma^t)_t$ where $\tau^t : M \to \Delta(A)$.
- Receiver plays only responsive strategies: $\tau^t(m_c) \neq \tau^t(m_q)$.

Expected Utility

- The stopping time S is a random variable which represents the period at which the receiver quits. (If the receiver never quits, then $S = \infty$.)
- The expected utility for the sender: $U_s(\sigma, \tau) = \mathbb{E}_{\sigma,\tau}[f^S(\theta^S)]$
- The expected utility for the receiver: $U_r(\sigma, \tau) = \mathbb{E}_{\sigma,\tau}[g^S(\theta^S)]$
- Expected utility from period t for the sender: $U_s^t(\sigma, \tau)$.
- Expected utility from period t for the receiver: $U_r^t(\sigma, \tau)$.

Regular strategy profile

- The sender's strategy σ is threshold strategy if for each t, there exists a threshold $\alpha^t \in [0, 1]$ such that $\sigma^t(\theta^t) = \begin{cases} m_c & \text{if } \theta^t \in [0, \alpha^t) \\ m_q & \text{if } \theta^t \in (\alpha^t, 1]. \end{cases}$
- The obeying strategy τ for the receiver is such that at each period t, $\tau^t(m_c) = a_c$ and $\tau^t(m_q) = a_q$.
- A threshold strategy σ for the sender is
  - stationary: if the thresholds at each period have the same value, that is, $\alpha^t = \alpha$ for all $t \geq 1$.
  - sincere: if it is the best response to the obeying strategy of the receiver.

- Regular strategy profile (σ, τ): σ is sincere and τ is obeying.

- **Interpretation:** The receiver simply obeys the sender and the sender plays the strategy which gives him the best possible payoff.
- Regular strategy profile is Essentially unique.

Results

Variants

Horizon:

- Finite horizon: $(T = 1, 2, 3, \ldots)$
- Infinite horizon: $(T = \infty)$

Payoffs:

- δ - Discounted payoffs: $(\delta < 1)$

  $f^t(\theta^t) = \delta^{t-1} \cdot f(\theta^t), \quad g^t(\theta^t) = \delta^{t-1} \cdot g(\theta^t)$

- Period independent payoffs: $(\delta = 1)$

  $f^t(\theta^t) = f(\theta^t), \quad g^t(\theta^t) = g(\theta^t)$

The solution concept: PBE

A strategy profile (σ, τ) is called a Perfect Bayesian Equilibrium (PBE) if for every period t, the following hold:

1. $U_s^t(\sigma, \tau) \geq U_s^t(\sigma', \tau)$ for every strategy σ'.
2. $U_r^t(\sigma, \tau) \geq U_r^t(\sigma, \tau')$ for every strategy τ'.

Results

Infinite horizon

Existence and uniqueness of stationary PBE

If the payoffs are δ-discounted with sufficiently high δ, the regular strategy profile is the unique PBE. Furthermore, this PBE is stationary.

Non Existence PBE

If the payoffs are period independent, there does not exist any PBE.

Finite horizon

Existence and uniqueness of PBE

If the payoffs are period independent or δ-discounted with sufficiently high δ, then the regular profile is the unique PBE.

Proof Idea: Backward induction !

Extensions and Conclusion

Multiple senders, 1 Receiver

- Set of Senders: $N = \{1, \ldots, n\}$.
- At each period t, each sender i sends a message m_i^t.

Existence of PBE

There exists a PBE in which

- Each sender plays the sincere strategy.
- At each period t, receiver chooses a number n_t.
- Receiver takes action a_q if $|\{i : m_i^t = m_q\}| \geq n_t$, otherwise action a_c.

Conclusion

- Typically, either the regular strategy profile is the unique PBE or there is no PBE. So in PBE:
  1) The receiver always follows the sender's recommendation.
  2) The sender obtains his best possible expected payoff.
- Multiple senders
  1) Uniqueness of PBE fails.
  2) Work in progress.

Example - Infinite horizon, discount factor δ

- $f^t(x) = \delta^{t-1} \cdot x^2$, $g^t(x) = \delta^{t-1} \cdot x$.
- If $\delta < 0.37$, then the game has no PBE.
- If $0.37 \leq \delta < 1$, then the game has unique PBE which is stationary.
- If $\delta = 1$, then the game has no PBE.

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