Dynamics and Coalitions in Sequential Games

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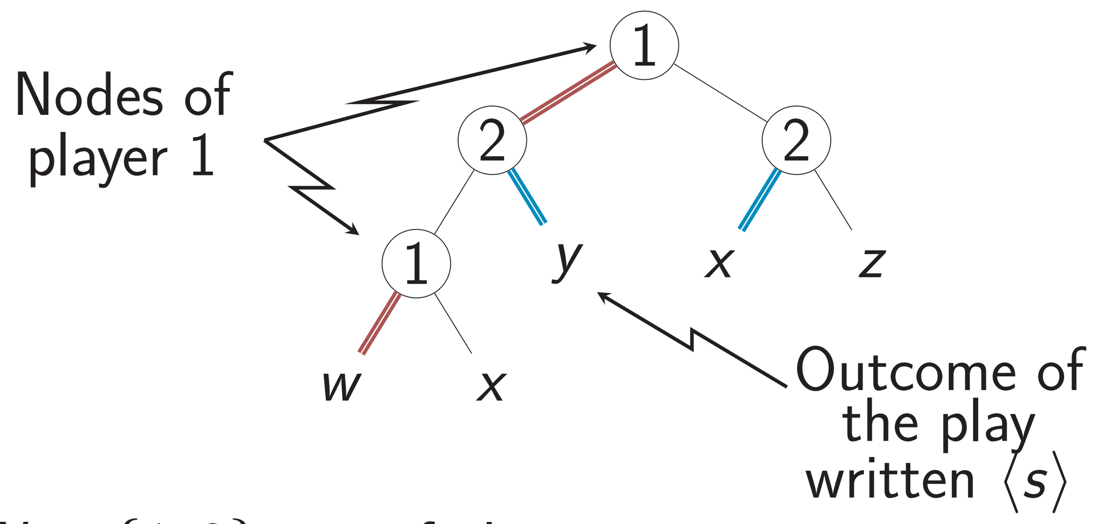
Finite Sequential Game



$N = \{1,2\}$, set of players

$O = \{w,x,y,z\}$, set of outcomes

|| Strategy of player 1, written s_1

|| Strategy of player 2, written s_2

(||, ||) Strategy Profile, written s

Preferences of players 1 and 2 :

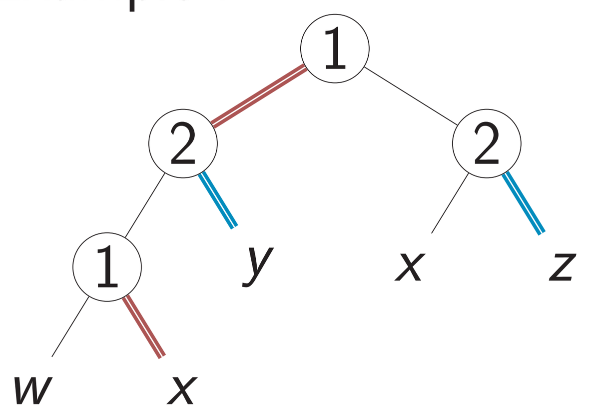
$x \prec_1 z \prec_1 y \prec_1 w$

$z \prec_2 w \prec_2 x \prec_2 y$

Nash Equilibrium

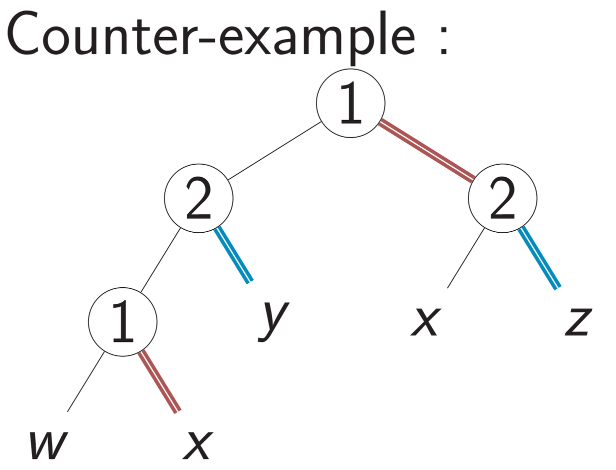
A strategy profile is a Nash Equilibrium (NE) if no player has any profitable deviation.

Example :



Player 1 can reach z but $z \prec_1 y$
Player 2 can reach x but $x \prec_2 y$

Counter-example :

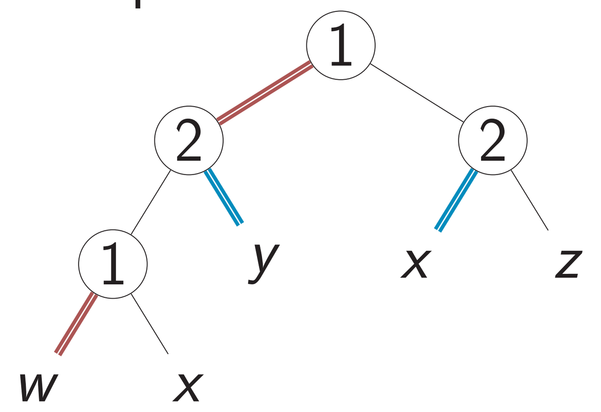


Player 1 can reach y instead of z, and $z \prec_1 y$.

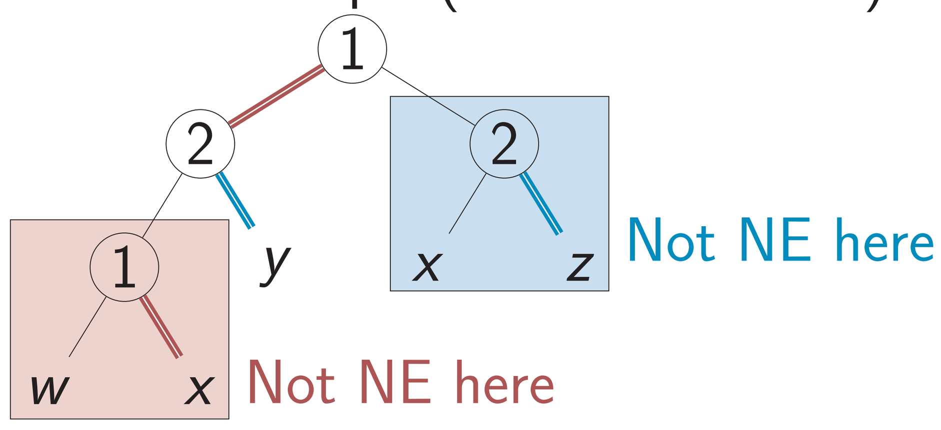
Subgame Perfect Equilibrium

A strategy profile is a Subgame Perfect Equilibrium (SPE) if it is an NE in every subgame.

Example :



Counter-example (NE but not SPE) :

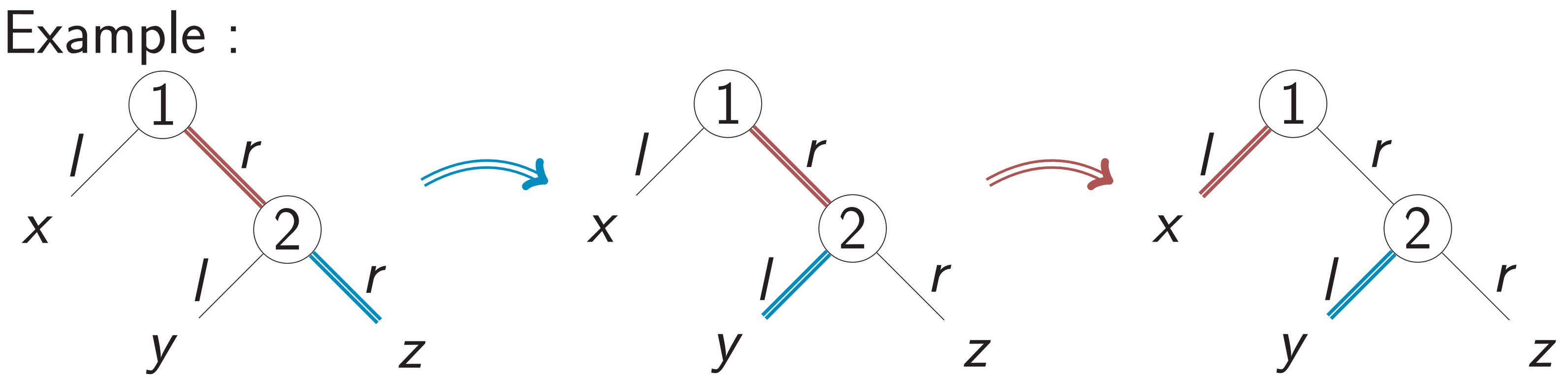


Dynamics

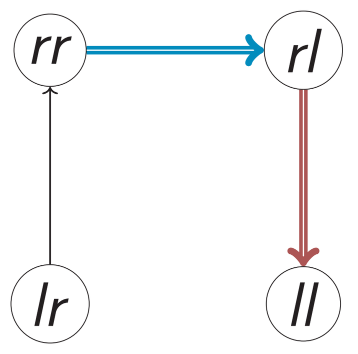
Idea :

- Begin with some strategy profile
- Let players improve their strategy (according to some conditions)
- Does this dynamics terminate ?
- If so, what are the terminal profiles ?

Example :



With $y \prec_1 x \prec_1 z$ and $x \prec_2 z \prec_2 y$.



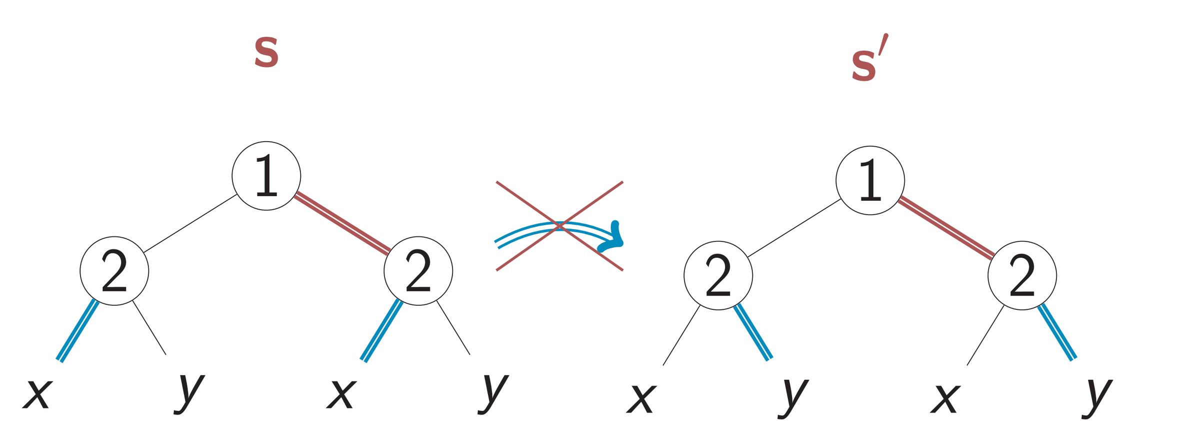
Terminal profile : NE

Lazy Improvement Dynamics

Definition

A couple of strategy profiles s, s' verifies the Lazy Improvement Dynamics if :

- Only one player changes his strategy.
- He improves his general payoff.
- Every change is along the play induced by s'.



With $x \prec_2 y$

This change is not allowed because player 2 makes a change on the left subgame that is not along the play induced by s'.

Theorem [LRP16, Corollary 11]

Let $N, O, (\prec_i)_{i \in N}$ be respectively a set of players, a set of outcomes and preferences. Then, the two following statements are equivalent:

1. In *all* games built over N, O, $(\prec_i)_{i \in N}$ the Lazy Improvement Dynamics terminates;
2. The preferences $(\prec_i)_{i \in N}$ are acyclic.

Moreover, the set of terminal profiles of the Lazy Improvement Dynamics is exactly the set of NEs of the game.

Subgame Improvement Dynamics

Definition

A couple of strategy profiles s, s' verifies the Subgame Improvement Dynamics if each player that changes his strategy improves it in the subgame rooted at the change.

Theorem [BGHR17, Theorem 4]

Let $N, O, (\prec_i)_{i \in N}$ be respectively a set of players, a set outcomes and preferences. Then, the two following statements are equivalent:

1. In *all* games built over $N, O, (\prec_i)_{i \in N}$ the Subgame Improvement Dynamics terminates;
2. The preferences $(\prec_i)_{i \in N}$ are acyclic.

Moreover, the set of terminal profiles of the Subgame Improvement Dynamics is exactly the set of SPEs of the game.

Coalitional Dynamics

Definition

A couple of strategy profiles s, s' verifies the Coalitional Improvement Dynamics if :

- Every player taking part to the coalition improves his general payoff.
- They play lazily (see third point of the definition of the Lazy Improvement Dynamics)

This Dynamics is quite more complicated and requires to study the structure of the preferences.

Theorem [BGHR17, Corollary 16]

Let $N, O, (\prec_i)_{i \in N}$ be respectively a set of players, a set of outcomes and a strict linear order modeling preferences. The following are equivalent:

(1) In *all* games built over $N, O, (\prec_i)_{i \in N}$ the Lazy Improvement Dynamics terminates;

(2) All games built over O, N and $(<_i)_{i \in N}$ admit a Strong Nash Equilibria;

(3) $(<_i)_{i \in N}$ is out of pattern for O.

[BGHR17] Thomas Brihaye, Gilles Geeraerts, Marion Hallet, and Stéphane Le Roux. Dynamics and coalitions in sequential games. In *Proceedings, GandALF 2017*, volume 256 of *EPTCS*, pages 136-150, 2017.

[LRP16] Stéphane Le Roux and Arno Pauly. A Semi-Potential for Finite and Infinite Sequential Games (Extended Abstract). In *Proceedings, GandALF 2016*, volume 226 of *EPTCS*, pages 242-256. Open Publishing Association, 2016.

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