TITLE AND ABSTRACT FOR TALK BY SALMA KUHLMANN

TITLE:

Primes and Irreducibles in Exponential Integer Parts of Ordered Exponential Fields (2 talks.

ABSTRACT:

An integer part (IP for short) Z of an ordered field F is a discretely ordered subring, with 1 as least positive element, and such that for every $x \in F$, there is a $z \in Z$ such that $z \leq x < z + 1$. Shepherdson [S] showed that IP's of real closed fields correspond to models of a fragment of Peano Arithmetic called Open Induction (OI for short). He used this observation to investigate the arithmetic properties of these rings. For example, he constructed an IP for the field of algebraic Puiseux series (with coefficients in the field of real algebraic numbers, and exponents in the group of rational numbers) with only standard irreducibles. In particular, the set of primes is not cofinal in this countable recursive model. On the other hand, subsequent to his work, several authors (e.g. [M], [B–O]) constructed such rings with unboundedly many infinite primes. Thus the "infinity of primes" is not provable from OI. In [M–R], the authors establish the existence of an IP for any real closed field. In [R], a proof for an exponential analogue is sketched: every exponential field (see [K]) has an *exponential integer part* (EIP for short; an EIP is an IP that satisfies moreover some closure conditions under the exponential function).

For fields of generalized power series $K = \mathbb{R}((G))$, an integer part is given by the ring $\mathbb{R}((G^{<0})) \oplus \mathbb{Z}$, the ring of series with negative exponents and integer constant term. In [B], the author characterizes irreducible elements in this ring by the order type of their support. In the Concluding Remarks, he asks whether this criterium for irreducibility applies in the presence of exponentiation. More precisely, one should investigate whether *every* EIP of an exponential field has unboundedly many irreducibles (primes).

In the first talk, we review the main notions and the background for the problem. In the second talk, we turn to results of [B–K–K] on EIP's of ordered exponential fields. Our main result is that *every exponential field has an EIP with cofinally many irreducibles*. We present two particulary interesting examples for which we get a stronger result: EIP's of *countable* exponential fields ([K; Chap. 1 Sect. 7]), and those of *Exponential- Logarithmic power series fields* ([K; Chap. 5 Sect. 2]). For the countable models (the "Exponential Algebraic Series Fields") described in [K; Example 1.45], we construct a "canonical" EIP. Based on generalizations of results of [B], we show that this EIP has cofinally many *primes with finite support*. Similarly, we construct a "canonical" EIP for the Exponential-Logarithmic power series fields. Generalizing results of [P], we show that this EIP has cofinally many *primes with infinite support*.

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