

## Games where you can play optimally with finite memory

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October 10, 2019

*GT ALGA annual meeting 2019*



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 *Work in progress* 

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*A sequel to the critically acclaimed blockbuster by Gimbert & Zielonka*

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GT ALGA a



Games where you can play optimally without any memory \*

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**Abstract.** Reactive systems are often modelled as two person antagonistic games where one player represents the system while his adversary represents the environment. Undoubtedly, the most popular games in this class are parity games and their cousins (Rabin, Streett and Muller). We also consider other types of payments, like the one used in economic control theory. This paper also discusses the use of automatic verification. The

## The talk in one slide

### Strategy synthesis for two-player turn-based games

Finding **good** controllers for systems interacting with an *antagonistic* environment.

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Two directions for **finite-memory determinacy**:

- 1 lifting under *objective combination* (with S. Le Roux and A. Pauly, in FSTTCS'18 [LPR18]),
- 2 complete characterization and *lifting from one-player games* (*ongoing work*).

- 1 Memoryless determinacy
- 2 Finite-memory determinacy and Boolean combinations
- 3 Characterization and lifting corollary
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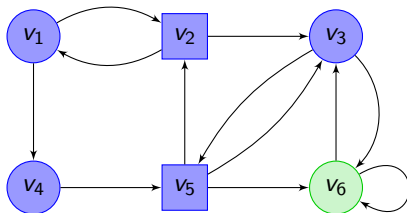
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# Two-player turn-based zero-sum games on graphs

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We consider *finite* arenas with vertex *colors* in  $C$ . Two players: circle ( $\mathcal{P}_1$ ) and square ( $\mathcal{P}_2$ ). Strategies  $C^* \times V_i \rightarrow V$ .

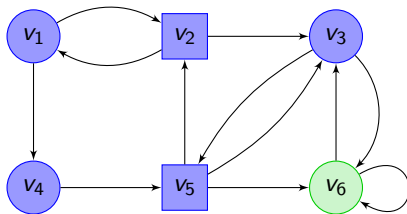
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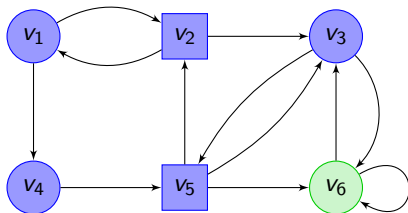


**From where can  $\mathcal{P}_1$  ensure to reach  $v_6$ ?**  
**How complex is his strategy?**

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Memoryless strategies ( $V_i \rightarrow V$ ) always suffice for reachability (for both players).

# When are memoryless strategies sufficient to play optimally?



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Virtually always for **simple** winning conditions!

Examples: reachability, safety, Büchi, parity, mean-payoff, energy, total-payoff, average-energy, etc.

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**Can we characterize when they are?**

Yes, thanks to Gimbert and Zielonka [[GZ05](#)].

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## Gimbert and Zielonka's characterization

Memoryless strategies suffice for a *preference relation*  $\sqsubseteq$  (and the induced winning conditions) **if and only if**

**1** it is **monotone**,

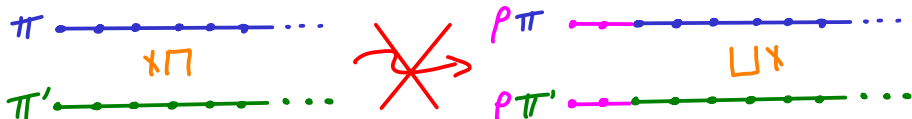
**2** it is **selective**.

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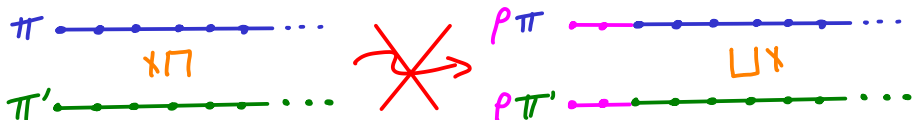
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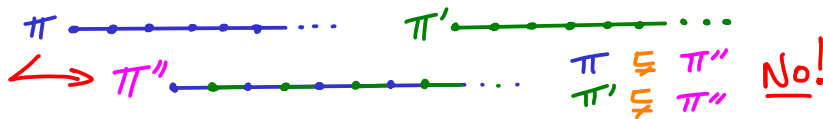
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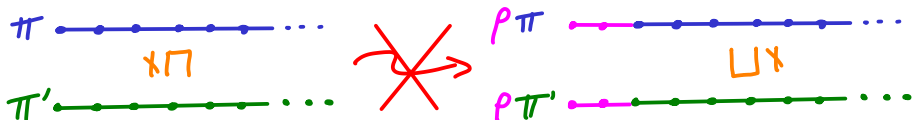


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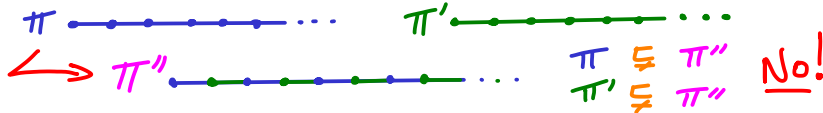
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Example: reachability.

⚠ No equivalent for FM ⚠

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then both players have optimal memoryless strategies **in all two-player arenas**.

*★ Extremely useful in practice! ★*

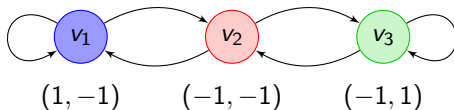
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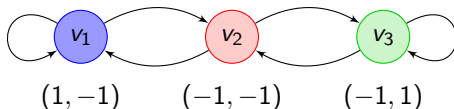


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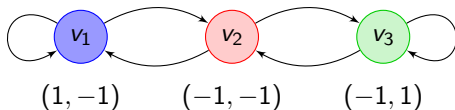


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### Two directions:

- 1 single-objective  $\rightsquigarrow$  multi-objective [LPR18],
- 2 GZ-like characterization and one-player  $\rightsquigarrow$  two-player.

1 Memoryless determinacy

2 Finite-memory determinacy and Boolean combinations

3 Characterization and lifting corollary

*With S. Le Roux  
& A. Paurdy, FSTTCS'18.*

4 Conclusion



## Combining winning conditions

### Our goal

We want a *general* and *abstract* theorem guaranteeing the sufficiency of **finite-memory strategies**<sup>a</sup> in games with **Boolean combinations of objectives** provided that the underlying **simple objectives** fulfill some criteria.

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### Drawbacks:

- ▷ concrete memory bounds are huge (as they depend on the most general upper bound).
- ▷ sufficient criterion, not full characterization.

## The building blocks

The full approach is technically involved but can be sketched intuitively.

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**⇒ Only one exception AFAWK (hSPE vs. opt. strategies).**

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  - ▷ We also have several **more precise results** (e.g., much lower bounds) for specific combinations and/or restrictive hypotheses.

Almost complete picture of the frontiers of FM determinacy for *combinations of objectives* but still **not a complete characterization à la Gimbert and Zielonka.**

- 1 Memoryless determinacy
- 2 Finite-memory determinacy and Boolean combinations
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With P. Boyer, S. Le Roux,  
Y. Oualhadj & P. Vandenhouve.  
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**Our dream:** *exact equivalent* in the finite-memory case.

## A partial counter-example (lifting corollary)

Let  $C \subseteq \mathbb{Z}$  and the winning condition for  $\mathcal{P}_1$  be

$$\overline{TP}(\pi) = \infty \quad \vee \quad \exists^{\infty} i \in \mathbb{N}, \sum_{i=0}^n c_i = 0$$

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**Both 1-player variants are finite-memory determined.**

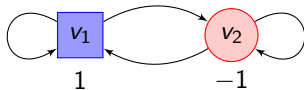


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↯ No exact  
equivalent  
to GZ ↯

But the two-player one is not!

⇒  $\mathcal{P}_1$  needs infinite memory to win.

*Hint:* non-monotony is a bigger threat in two-player games.  
In one-player games, *finite* memory may help.

## A new hope

Our goal

GZ-like characterization for finite-memory strategies.

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⇒ Intuitively, selectivity *modulo a memory skeleton*.

**We obtain a natural GZ-equivalent for FM determinacy, including the lifting corollary (1-p. to 2-p.)!**

Still some elements to flesh out.  
⇒ Preprint writing in progress.

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## Two directions

### Combinations of objectives

- ▷ Matches our current knowledge almost-exactly.
- ▷ Useful when the underlying obj. are well-understood.
- ▷ With Le Roux and Pauly [[LPR18](#)] (on [arXiv](#)).

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Understand and characterize the frontiers of FM-determinacy.

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### GZ-like criterion

- ▷ No exact equivalent.
- ▷ Natural criterion and useful lifting corollary.
- ▷ With Bouyer, Le Roux, Oualhadj and Vandenhove, **ongoing work.**

Thank you! Any question?

# References I



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