

Rich Behavioral Models: Illustration on Journey Planning

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The talk in one slide

Strategy synthesis for Markov Decision Processes (MDPs)

Finding **good** controllers for systems interacting with a *stochastic* environment.

- Good? Performance evaluated through *payoff functions*.
- Usual problem is to optimize the *expected performance* or the *probability of achieving a given performance level*.
- Not sufficient for many practical applications.
 - ▷ Several extensions, more expressive but also more complex...

Aim of this survey talk

Give a flavor of classical questions and extensions (*rich behavioral models*), illustrated on the stochastic shortest path (*SSP*).

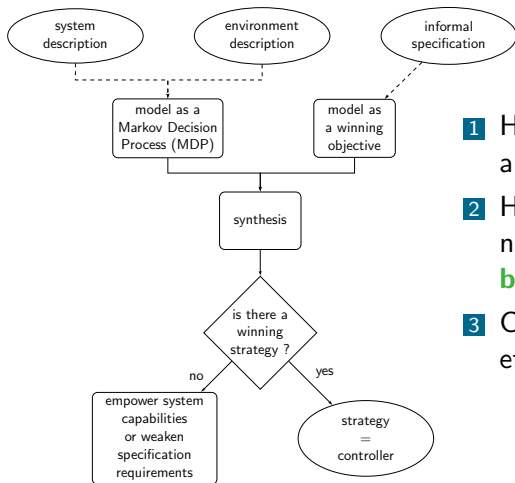
- 1 Context, MDPs, strategies
- 2 Classical stochastic shortest path problems
- 3 Good expectation under acceptable worst-case
- 4 Percentile queries in multi-dimensional MDPs
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Multi-criteria quantitative synthesis

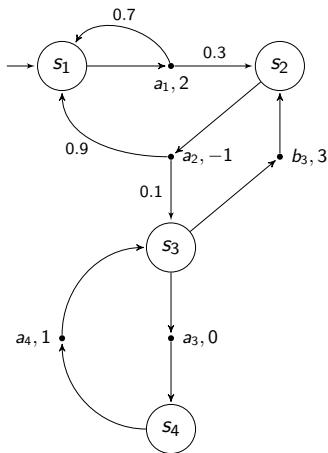
- Verification and synthesis:
 - ▷ a reactive **system** to *control*,
 - ▷ an *interacting environment*,
 - ▷ a **specification** to *enforce*.
- Model of the (discrete) interaction?
 - ▷ Antagonistic environment: 2-player game on graph.
 - ▷ **Stochastic environment: MDP.**
- **Quantitative** specifications. Examples:
 - ▷ Reach a state s before x time units \rightsquigarrow shortest path.
 - ▷ Minimize the average response-time \rightsquigarrow mean-payoff.
- Focus on **multi-criteria quantitative models**
 - ▷ to reason about *trade-offs* and *interplays*.

Strategy (policy) synthesis for MDPs



- 1 How complex is it to **decide** if a winning strategy exists?
- 2 How complex such a **strategy** needs to be? **Simpler is better.**
- 3 Can we **synthesize** one efficiently?

Markov decision processes



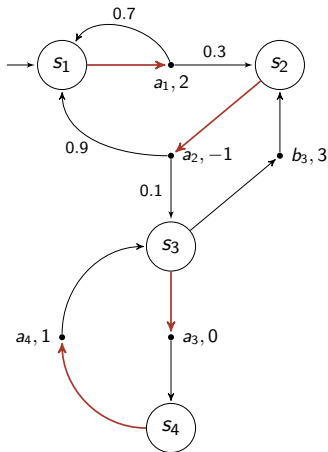
- MDP $D = (S, s_{\text{init}}, A, \delta, w)$.
 - ▷ Finite sets of states S and actions A ,
 - ▷ probabilistic transition $\delta: S \times A \rightarrow \mathcal{D}(S)$,
 - ▷ weight function $w: A \rightarrow \mathbb{Z}$.
- **Run** (or play): $\rho = s_1 a_1 \dots a_{n-1} s_n \dots$ such that $\delta(s_i, a_i, s_{i+1}) > 0$ for all $i \geq 1$.
 - ▷ Set of runs $\mathcal{R}(D)$.
 - ▷ Set of histories (finite runs) $\mathcal{H}(D)$.
- **Strategy** $\sigma: \mathcal{H}(D) \rightarrow \mathcal{D}(A)$.
 - ▷ $\forall h$ ending in s , $\text{Supp}(\sigma(h)) \in A(s)$.

Markov decision processes

Sample *pure memoryless* strategy σ .

Sample run $\rho = s_1 a_{1,2} s_2 a_{2,2} s_1 a_1 s_2 a_2 (s_3 a_3 s_4 a_4)^\omega$.

Other possible run $\rho' = s_1 a_1 s_2 a_2 (s_3 a_3 s_4 a_4)^\omega$.



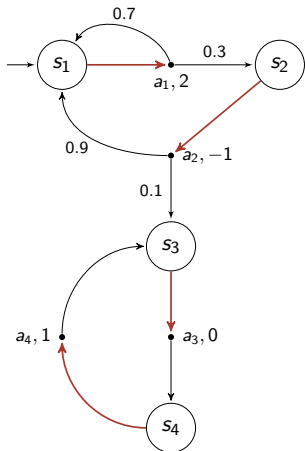
- Strategies may use

- ▷ finite or infinite **memory**,
- ▷ **randomness**.

- Payoff functions** map runs to numerical values:

- ▷ truncated sum up to $T = \{s_3\}$:
 $TS^T(\rho) = 2$, $TS^T(\rho') = 1$,
- ▷ mean-payoff: $\underline{MP}(\rho) = \underline{MP}(\rho') = 1/2$,
- ▷ many more.

Markov chains



Once strategy σ fixed, fully stochastic process:
 \rightsquigarrow **Markov chain (MC)** M .

State space = product of the MDP and the memory of σ .

- Event $\mathcal{E} \subseteq \mathcal{R}(M)$
 - ▷ probability $\mathbb{P}_M(\mathcal{E})$
- Measurable $f: \mathcal{R}(M) \rightarrow \mathbb{R} \cup \{\infty\}$,
 - ▷ expected value $\mathbb{E}_M(f)$

Aim of this survey

Compare different types of quantitative specifications for MDPs

- ▷ w.r.t. the complexity of the decision problem,
- ▷ w.r.t. the complexity of winning strategies.

Recent extensions share a common philosophy: framework for the synthesis of strategies with *richer performance guarantees*.

- ▷ Our work deals with many different payoff functions.

Focus on the **shortest path problem** in this talk.

- ▷ Not the most involved technically, natural applications.
- ↪ Useful to understand the **practical interest** of each variant.

Joint work with R. Berthon, V. Bruyère, E. Filiot, J.-F. Raskin, O. Sankur [BFRR17, RRS17, RRS15, BCH⁺16, Ran16, BRR17].

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Stochastic shortest path

Shortest path problem for *weighted graphs*

Given state $s \in S$ and target set $T \subseteq S$, find a path from s to a state $t \in T$ that minimizes the sum of weights along edges.

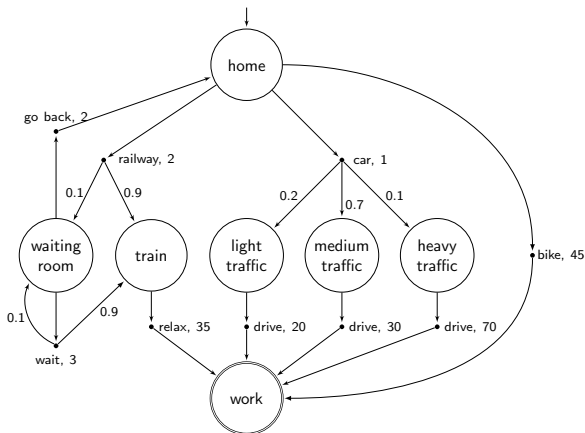
- ▶ PTIME algorithms (Dijkstra, Bellman-Ford, etc) [CGR96].

We focus on MDPs with **strictly positive weights** for the SSP.

- ▶ **Truncated sum** payoff function for $\rho = s_1 a_1 s_2 a_2 \dots$ and target set T :

$$\text{TS}^T(\rho) = \begin{cases} \sum_{j=1}^{n-1} w(a_j) & \text{if } s_n \text{ first visit of } T, \\ \infty & \text{if } T \text{ is never reached.} \end{cases}$$

Planning a journey in an uncertain environment



Each action takes **time**, target = work.

- ▶ What kind of **strategies** are we looking for when the environment is stochastic?

SSP-E: minimizing the expected length to target

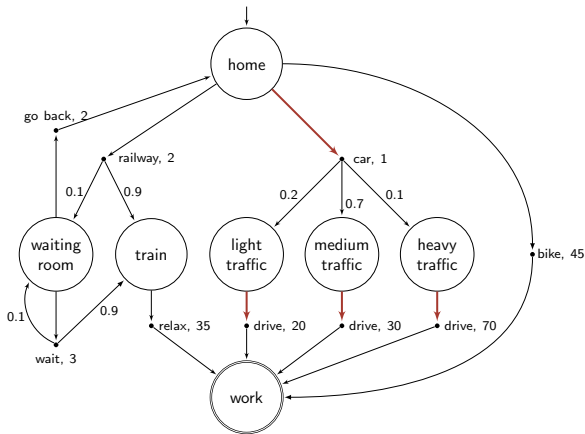
SSP-E problem

Given MDP $D = (S, s_{\text{init}}, A, \delta, w)$, target set T and threshold $\ell \in \mathbb{Q}$, decide if there exists σ such that $\mathbb{E}_D^\sigma(\text{TS}^T) \leq \ell$.

Theorem [BT91]

The SSP-E problem can be decided in **polynomial time**. Optimal **pure memoryless** strategies always exist and can be constructed in polynomial time.

SSP-E: illustration



- ▷ Pure memoryless strategies suffice.
- ▷ Taking the **car** is optimal: $\mathbb{E}_D^\sigma(\text{TS}^T) = 33$.

SSP-E: PTIME algorithm

1 Graph analysis (linear time):

- ▷ s not connected to $T \Rightarrow \infty$ and remove,
- ▷ $s \in T \Rightarrow 0$.

2 Linear programming (LP, polynomial time).

For each $s \in S \setminus T$, one variable x_s ,

$$\max \sum_{s \in S \setminus T} x_s$$

under the constraints

$$x_s \leq w(a) + \sum_{s' \in S \setminus T} \delta(s, a, s') \cdot x_{s'} \quad \text{for all } s \in S \setminus T, \text{ for all } a \in A(s).$$

SSP-E: PTIME algorithm

1 Graph analysis (linear time):

- ▷ s not connected to $T \Rightarrow \infty$ and remove,
- ▷ $s \in T \Rightarrow 0$.

2 Linear programming (LP, polynomial time).

Optimal solution \mathbf{v} :

↪ \mathbf{v}_s = expectation from s to T under an optimal strategy.

Optimal pure memoryless strategy $\sigma^{\mathbf{v}}$:

$$\sigma^{\mathbf{v}}(s) = \arg \min_{a \in A(s)} \left[w(a) + \sum_{s' \in S \setminus T} \delta(s, a, s') \cdot \mathbf{v}_{s'} \right].$$

↪ **Playing optimally = locally optimizing present + future.**

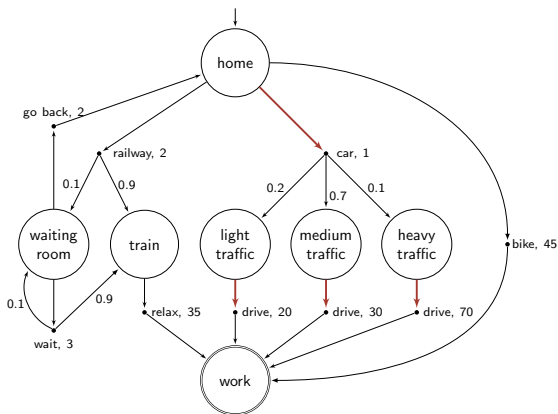
SSP-E: PTIME algorithm

- 1 Graph analysis (linear time):
 - ▷ s not connected to $T \Rightarrow \infty$ and remove,
 - ▷ $s \in T \Rightarrow 0$.
- 2 **Linear programming (LP)**, polynomial time).

In practice, **value and strategy iteration** algorithms often used:

- ▷ best performance in most cases but **exponential** in the worst-case,
- ▷ fixed point algorithms, successive solution improvements [BT91, dA99, HM14].

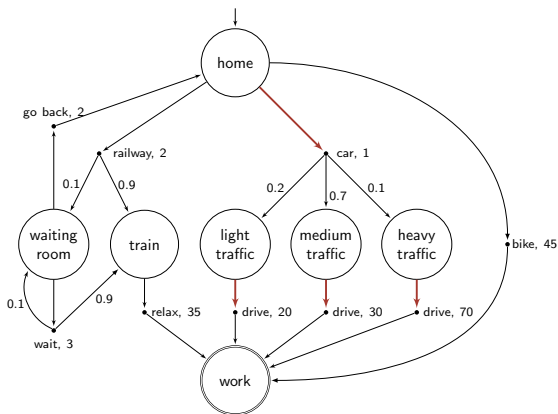
Traveling without taking too many risks



Minimizing the *expected time* to destination makes sense **if** we travel often and it is not a problem to be late.

With car, in 10% of the cases, the journey takes 71 minutes.

Traveling without taking too many risks



Most bosses will not be happy if we are late too often...

~> what if we are risk-averse and want to avoid that?

SSP-P: forcing short paths with high probability

SSP-P problem

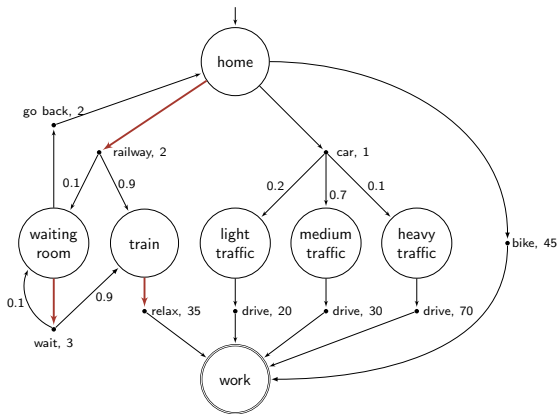
Given MDP $D = (S, s_{\text{init}}, A, \delta, w)$, target set T , threshold $\ell \in \mathbb{N}$, and probability threshold $\alpha \in [0, 1] \cap \mathbb{Q}$, decide if there exists a strategy σ such that $\mathbb{P}_D^\sigma[\{\rho \in \mathcal{R}_{s_{\text{init}}}(D) \mid \text{TS}^T(\rho) \leq \ell\}] \geq \alpha$.

Theorem

The SSP-P problem can be decided in **pseudo-polynomial time**, and it is **PSPACE-hard**. Optimal **pure strategies with pseudo-polynomial memory** always exist and can be constructed in pseudo-polynomial time.

See [HK15] for hardness and for example [RRS17] for algorithm.

SSP-P: illustration



Specification: reach work within 40 minutes with 0.95 probability

Sample strategy: take the **train** $\rightsquigarrow \mathbb{P}_D^\sigma[\text{TS}^{\text{work}} \leq 40] = 0.99$

Bad choices: car (0.9) and bike (0.0)

SSP-P: pseudo-PTIME algorithm (1/2)

Key idea: pseudo-PTIME reduction to the **stochastic reachability problem (SR)**

SR problem

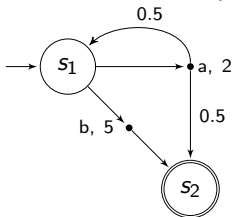
Given unweighted MDP $D = (S, s_{\text{init}}, A, \delta)$, target set T and probability threshold $\alpha \in [0, 1] \cap \mathbb{Q}$, decide if there exists a strategy σ such that $\mathbb{P}_D^\sigma[\diamond T] \geq \alpha$.

Theorem

The SR problem can be decided in **polynomial time**. Optimal **pure memoryless strategies** always exist and can be constructed in polynomial time.

- ▶ Linear programming (similar to SSP-E).

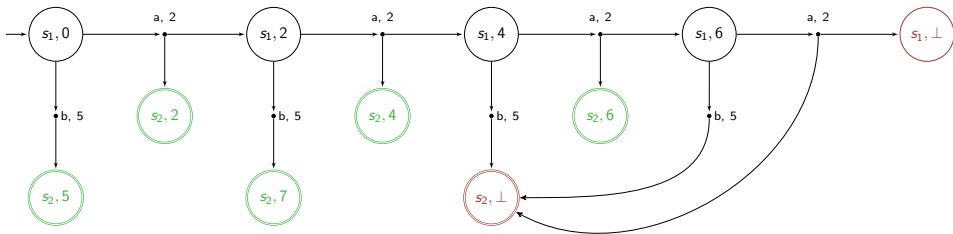
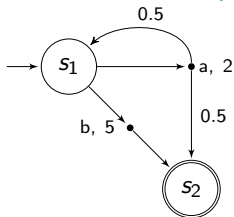
SSP-P: pseudo-PTIME algorithm (2/2)



Sketch of the reduction:

- 1 Start from D , $T = \{s_2\}$, and $\ell = 7$.
- 2 Build D_ℓ by unfolding D , tracking the current sum *up to the threshold* ℓ , and integrating it in the states of the expanded MDP.

SSP-P: pseudo-PTIME algorithm (2/2)



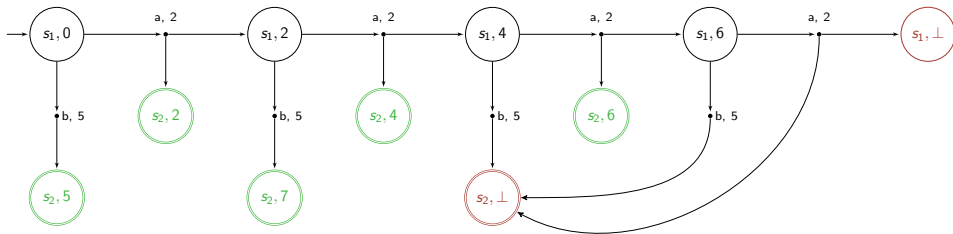
SSP-P: pseudo-PTIME algorithm (2/2)

- 3 Relation between runs of D and D_ℓ :

$$TS^T(\rho) \leq \ell \iff \rho' \models \diamond T', T' = T \times \{0, 1, \dots, \ell\}.$$

- 4 Solve the SR problem on D_ℓ .

- ▷ Memoryless strategy in $D_\ell \rightsquigarrow$ pseudo-polynomial memory in D in general.



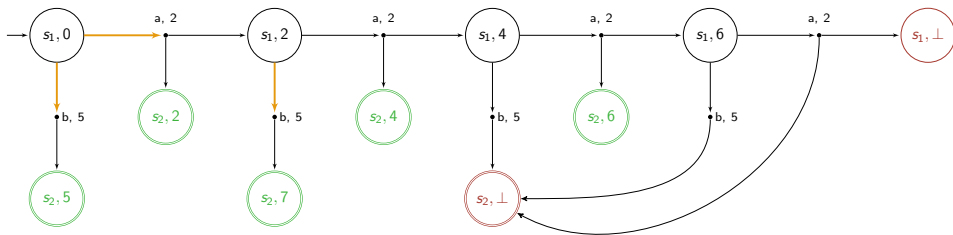
SSP-P: pseudo-PTIME algorithm (2/2)

If we just want to minimize the risk of exceeding $\ell = 7$,

- ▷ an obvious possibility is to play b directly,
- ▷ playing a only once is also acceptable.

For the SSP-P problem, **both strategies are equivalent.**

~ We need richer models to discriminate them!

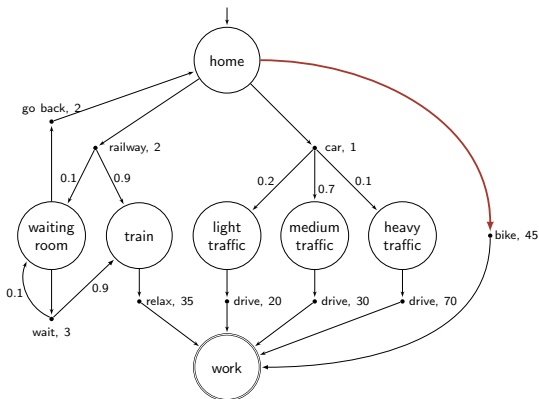


Related work (non-exhaustive)

- SSP-P problem with relaxed hypotheses [Oht04, SO13].
- SSP-E problem with relaxed hypotheses [BBD⁺18].
- *Quantile queries* [UB13]: minimizing the value ℓ of an SSP-P problem for some fixed α . Extended to *cost problems* [HK15, HKL17].
- SSP-E problem in **multi-dimensional** MDPs [FKN⁺11].

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SP-G: strict worst-case guarantees

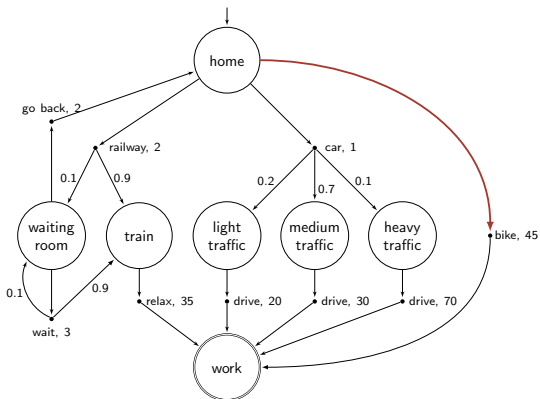


Specification: *guarantee* that work is reached within 60 minutes (to avoid missing an important meeting).

Sample strategy: take the **bike** $\rightsquigarrow \forall \rho \in \text{Out}_D^\sigma: \text{TS}^{\text{work}}(\rho) \leq 60$.

Bad choices: train ($wc = \infty$) and car ($wc = 71$).

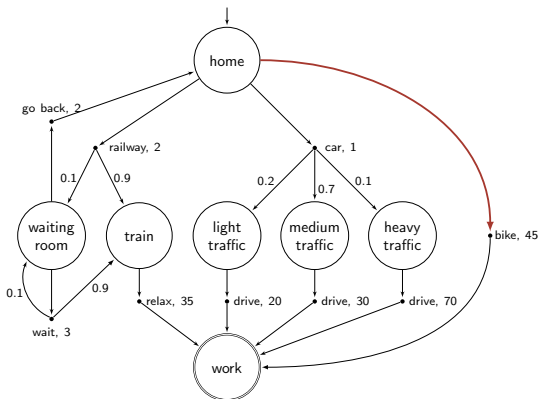
SP-G: strict worst-case guarantees



Winning **surely (worst-case)** \neq **almost-surely (proba. 1)**.

- ▶ Train ensures reaching work with probability one, but does not prevent runs where work is never reached.

SP-G: strict worst-case guarantees



Worst-case analysis \rightsquigarrow **two-player game** against an antagonistic adversary.

- ▶ Forget about probabilities and give the choice of transitions to the adversary.

SP-G: shortest path game problem

SP-G problem

Given MDP $D = (S, s_{\text{init}}, A, \delta, w)$, target set T and threshold $\ell \in \mathbb{N}$, decide if there exists a strategy σ such that for all $\rho \in \text{Out}_D^\sigma$, we have that $\text{TS}^T(\rho) \leq \ell$.

Theorem [KBB⁺08]

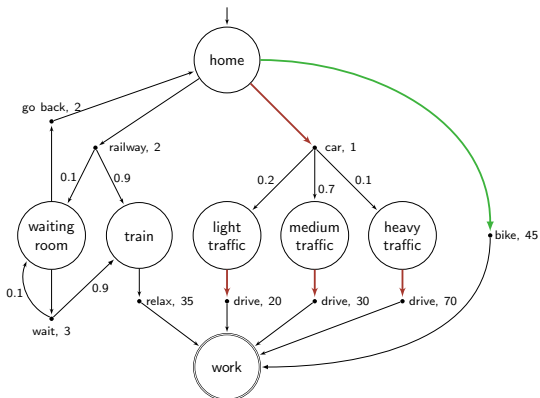
The SP-G problem can be decided in **polynomial time**. Optimal **pure memoryless** strategies always exist and can be constructed in polynomial time.

- ▷ Dynamic programming.

Related work (non-exhaustive)

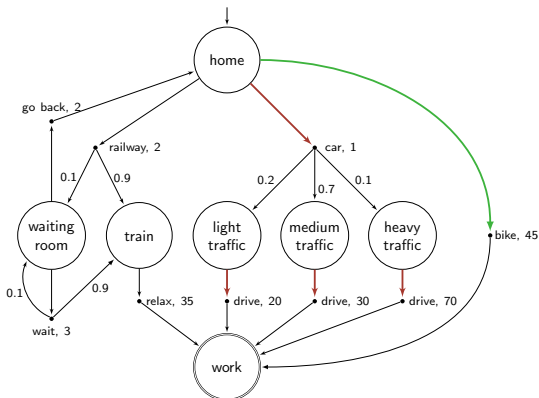
- Pseudo-PTIME for arbitrary weights [BGHM17, FGR15].
- Arbitrary weights + multiple dimensions \rightsquigarrow undecidable (by adapting the proof of [CDRR15] for total-payoff).

SSP-WE = SP-G \cap SSP-E - illustration



- SSP-E: **car** \rightsquigarrow $\mathbb{E} = 33$ but **wc** = 71 > 60
- SP-G: **bike** \rightsquigarrow **wc** = 45 < 60 but $\mathbb{E} = 45 \gg \gg 33$

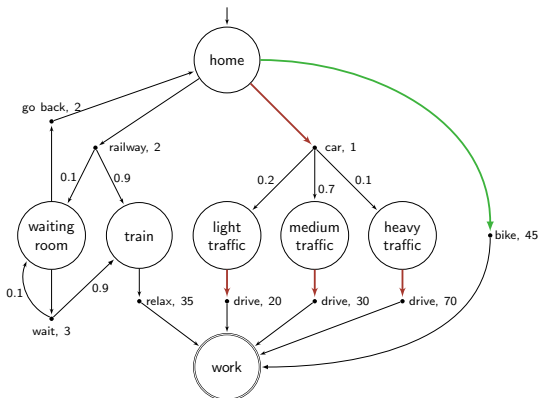
SSP-WE = SP-G \cap SSP-E - illustration



Can we do better?

- ▶ **Beyond worst-case synthesis** [BFRR17]: minimize the expected time under the worst-case constraint.

SSP-WE = SP-G \cap SSP-E - illustration



Sample strategy: try train up to 3 delays then switch to bike.

$\rightsquigarrow wc = 58 < 60$ and $\mathbb{E} \approx 37.34 \ll 45$

\rightsquigarrow pure *finite-memory* strategy

SSP-WE: beyond worst-case synthesis

SSP-WE problem

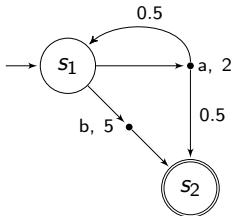
Given MDP $D = (S, s_{\text{init}}, A, \delta, w)$, target set T , and thresholds $\ell_1 \in \mathbb{N}$, $\ell_2 \in \mathbb{Q}$, decide if there exists a strategy σ such that:

- 1 $\forall \rho \in \text{Out}_D^\sigma: \text{TS}^T(\rho) \leq \ell_1,$
- 2 $\mathbb{E}_D^\sigma(\text{TS}^T) \leq \ell_2.$

Theorem [BFRR17]

The SSP-WE problem can be decided in **pseudo-polynomial time** and is **NP-hard**. **Pure pseudo-polynomial-memory** strategies are always sufficient and in general necessary, and satisfying strategies can be constructed in pseudo-polynomial time.

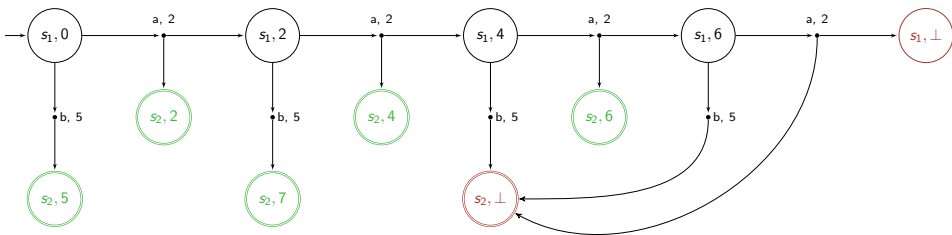
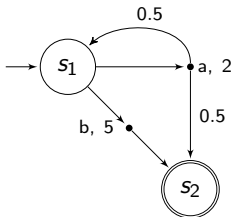
SSP-WE: pseudo-PTIME algorithm



Consider SSP-WE problem for $\ell_1 = 7$ (wc), $\ell_2 = 4.8$ (\mathbb{E}).

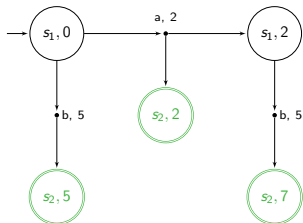
- ▶ Reduction to the SSP-E problem on a pseudo-polynomial-size expanded MDP.
- 1** Build unfolding as for SSP-P problem w.r.t. worst-case threshold ℓ_1 .

SSP-WE: pseudo-PTIME algorithm



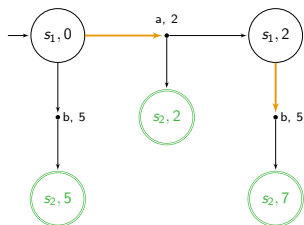
SSP-WE: pseudo-PTIME algorithm

- 2 Compute R , the attractor of $T' = T \times \{0, 1, \dots, \ell_1\}$.
- 3 Restrict MDP to $D' = D_{\ell_1} \downarrow R$, the safe part w.r.t. SP-G.



SSP-WE: pseudo-PTIME algorithm

- 4 Compute **memoryless optimal strategy** σ in D' for SSP-E.
- 5 Answer is YES iff $\mathbb{E}_{D'}^{\sigma}(\text{TS}^{T'}) \leq \ell_2$.



Here,
 $\mathbb{E}_{D'}^{\sigma}(\text{TS}^{T'}) = 9/2$.

SSP-WE: wrap-up

SSP	complexity	strategy
SSP-E	PTIME	pure memoryless
SSP-P	pseudo-PTIME / PSPACE-h.	pure pseudo-poly.
SSP-G	PTIME	pure memoryless
SSP-WE	pseudo-PTIME / NP-h.	pure pseudo-poly.

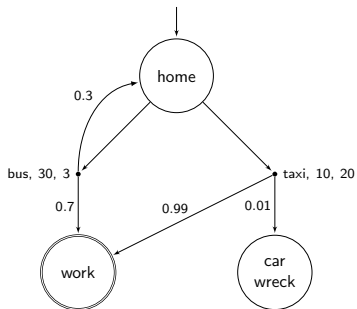
- ▶ NP-hardness \Rightarrow inherently harder than SSP-E and SSP-G.

Related work (non-exhaustive)

- BWC synthesis problems for mean-payoff [BFRR17] and parity [BRR17] belong to $NP \cap coNP$. Much more involved technically.
 - ⇒ Additional modeling power for free w.r.t. worst-case problems.
- Multi-dimensional extension for mean-payoff [CR15].
- Integration of BWC concepts in UPPAAL [DJL⁺14].
- Optimizing the expected mean-payoff under energy constraints [BKN16] or Boolean constraints [AKV16].
- Recent extensions to POMDPs [CNP⁺17, KPR18, CENR18].
- Conditional value-at-risk [KM18].

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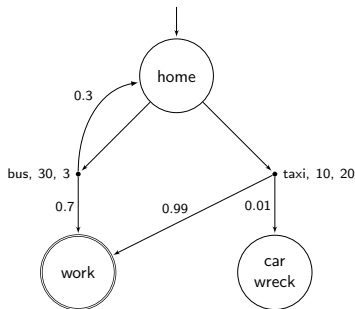
Multiple objectives \implies trade-offs



Two-dimensional weights on actions: *time* and *cost*.

Often necessary to consider **trade-offs**: e.g., between the probability to reach work in due time and the risks of an expensive journey.

Multiple objectives \implies trade-offs

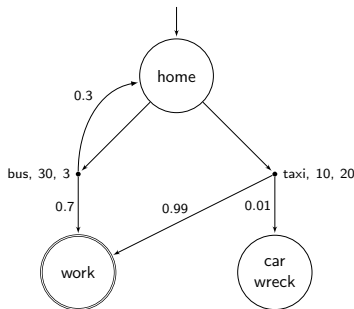


SSP-P problem considers a **single percentile constraint**.

- **C1**: 80% of runs reach work in at most 40 minutes.
 - ▷ Taxi $\rightsquigarrow \leq 10$ minutes with probability $0.99 > 0.8$.
- **C2**: 50% of them cost at most 10\$ to reach work.
 - ▷ Bus $\rightsquigarrow \geq 70\%$ of the runs reach work for 3\$.

Taxi $\not\models$ C2, bus $\not\models$ C1. What if we want $C1 \wedge C2$?

Multiple objectives \implies trade-offs

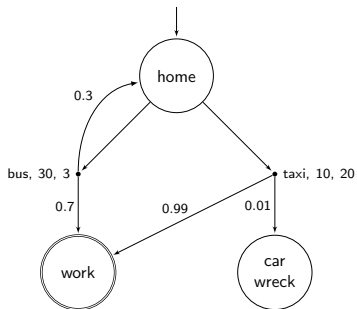


- **C1:** 80% of runs reach work in at most 40 minutes.
- **C2:** 50% of them cost at most 10\$ to reach work.

Study of **multi-constraint percentile queries** [RRS17].

- ▷ Sample strategy: bus once, then taxi. Requires *memory*.
- ▷ Another strategy: bus with probability 3/5, taxi with probability 2/5. Requires *randomness*.

Multiple objectives \implies trade-offs



- **C1:** 80% of runs reach work in at most 40 minutes.
- **C2:** 50% of them cost at most 10\$ to reach work.

Study of **multi-constraint percentile queries** [RRS17].

In general, *both memory and randomness* are required.

\neq **Previous problems.**

SSP-PQ: multi-constraint percentile queries (1/2)

SSP-PQ problem

Given d -dimensional MDP $D = (S, s_{\text{init}}, A, \delta, w)$, and $q \in \mathbb{N}$ percentile constraints described by target sets $T_i \subseteq S$, dimensions $k_i \in \{1, \dots, d\}$, value thresholds $\ell_i \in \mathbb{N}$ and probability thresholds $\alpha_i \in [0, 1] \cap \mathbb{Q}$, where $i \in \{1, \dots, q\}$, decide if there exists a strategy σ such that query \mathcal{Q} holds, with

$$\mathcal{Q} := \bigwedge_{i=1}^q \mathbb{P}_D^\sigma [\text{TS}_{k_i}^{T_i} \leq \ell_i] \geq \alpha_i,$$

where $\text{TS}_{k_i}^{T_i}$ denotes the truncated sum on dimension k_i and w.r.t. target set T_i .

Very general framework: multiple constraints related to \neq dimensions, and \neq target sets \implies great flexibility in modeling.

SSP-PQ: multi-constraint percentile queries (2/2)

Theorem [RRS17]

The SSP-PQ problem can be decided in

- **exponential time** in general,
- **pseudo-polynomial time** for single-dimension single-target multi-constraint queries.

It is **PSPACE-hard** even for single-constraint queries. **Randomized exponential-memory** strategies are always sufficient and in general necessary, and satisfying strategies can be constructed in exponential time.

- ▷ Unfolding + multiple reachability problem [EKVY08, RRS17].
- ▷ PSPACE-hardness already true for SSP-P [HK15].
- ↪ SSP-PQ = wide extension for **basically no price in complexity**.

SSP-PQ: wrap-up

SSP	complexity	strategy
SSP-E	PTIME	pure memoryless
SSP-P	pseudo-PTIME / PSPACE-h.	pure pseudo-poly.
SSP-G	PTIME	pure memoryless
SSP-WE	pseudo-PTIME / NP-h.	pure pseudo-poly.
SSP-PQ	EXPTIME (p.-PTIME) / PSPACE-h.	randomized exponential

- ▶ SSP-PQ is undecidable for arbitrary weights in multi-dimensional MDPs, even with a unique target set [RRS17].
- ▶ Clever unfolding technique in [HJKQ18].

Percentile queries: overview (1/2)

■ Wide range of payoff functions

- ▷ multiple reachability,
- ▷ inf, sup, lim inf, lim sup,
- ▷ mean-payoff ($\overline{\text{MP}}$, $\underline{\text{MP}}$),
- ▷ shortest path (SP),
- ▷ discounted sum (DS).

■ Several variants:

- ▷ multi-dim. multi-constraint,
- ▷ single-dim. multi-constraint,
- ▷ single-constraint.

■ For each one:

- ▷ algorithms,
- ▷ lower bounds,
- ▷ memory requirements.

↪ **Complete picture** for this new framework.

Percentile queries: overview (2/2)

	Single-constraint	Single-dim. Multi-constraint	Multi-dim. Multi-constraint
Reachability	P [Put94]	$P(D) \cdot E(Q)$ [EKVY08], PSPACE-h	—
$f \in \mathcal{F}$	P [CH09]	P	$P(D) \cdot E(Q)$ PSPACE-h.
\overline{MP}	P [Put94]	P	P
\underline{MP}	P [Put94]	$P(D) \cdot E(Q)$	$P(D) \cdot E(Q)$
SP	$P(D) \cdot P_{ps}(Q)$ [HK15] PSPACE-h. [HK15]	$P(D) \cdot P_{ps}(Q)$ (one target) PSPACE-h. [HK15]	$P(D) \cdot E(Q)$ PSPACE-h. [HK15]
ε -gap DS	$P_{ps}(D, Q, \varepsilon)$ NP-h.	$P_{ps}(D, \varepsilon) \cdot E(Q)$ NP-h.	$P_{ps}(D, \varepsilon) \cdot E(Q)$ PSPACE-h.

- ▷ $\mathcal{F} = \{\text{inf}, \text{sup}, \text{lim inf}, \text{lim sup}\}$
- ▷ $D = \text{model size}, Q = \text{query size}$
- ▷ $P(x), E(x)$ and $P_{ps}(x)$ resp. denote polynomial, exponential and pseudo-polynomial time in parameter x .

All results without reference are established in [RRS17].

Percentile queries: overview (2/2)

	Single-constraint	Single-dim. Multi-constraint	Multi-dim. Multi-constraint
Reachability	P [Put94]	$P(D) \cdot E(Q)$ [EKVY08], PSPACE-h	—
$f \in \mathcal{F}$	P [CH09]	P	$P(D) \cdot E(Q)$ PSPACE-h.
\overline{MP}	P [Put94]	P	P
\underline{MP}	P [Put94]	$P(D) \cdot E(Q)$	$P(D) \cdot E(Q)$
SP	$P(D) \cdot P_{ps}(Q)$ [HK15] PSPACE-h. [HK15]	$P(D) \cdot P_{ps}(Q)$ (one target) PSPACE-h. [HK15]	$P(D) \cdot E(Q)$ PSPACE-h. [HK15]
ε -gap DS	$P_{ps}(D, Q, \varepsilon)$ NP-h.	$P_{ps}(D, \varepsilon) \cdot E(Q)$ NP-h.	$P_{ps}(D, \varepsilon) \cdot E(Q)$ PSPACE-h.

In most cases, only **polynomial in the model size**.

- ▶ In practice, the query size can often be bounded while the model can be very large.

- 1 Context, MDPs, strategies
- 2 Classical stochastic shortest path problems
- 3 Good expectation under acceptable worst-case
- 4 Percentile queries in multi-dimensional MDPs
- 5 Conclusion**

Summary: stochastic shortest path problem

- **SSP-E:** minimize the expected sum to target.
 - ▷ Actual outcomes may vary greatly.
- **SSP-P:** maximize the probability of acceptable performance.
 - ▷ No control over the quality of bad runs, no average-case performance.
- **SP-G:** maximize the worst-case performance, extreme risk-aversion.
 - ▷ Strict worst-case guarantees, no average-case performance.
- **SSP-WE:** $SSP-E \cap SP-G$.
 - ▷ Based on **beyond worst-case synthesis** [BFRR17].
- **SSP-PQ:** extends SSP-P to **multi-constraint percentile queries** [RRS17].
 - ▷ Multi-dimensional, flexible, trade-offs.
 - ▷ Complexity usually acceptable w.r.t. model size.

Rich behavioral models: challenges

1 Plethora of theoretical models.

- ▷ Fundamental question: identify and understand the common core, advance toward unification.
- ▷ Can be an obstacle to adoption by practitioners.

2 Practical applicability.

- ▷ Efficiency must be increased (e.g., by using learning techniques).
- ▷ Tool support is key.

If you are interested...

... **consider attending MoRe 2018**, the 1st International Workshop on Multi-objective Reasoning in Verification and Synthesis, to be held in Oxford (FLoC 2018), on July 13.

Thank you! Any question?

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