Looking at Mean-Payoff and Total-Payoff through Windows

K. Chatterjee (IST Austria) L. Doyen (ENS Cachan)
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20.09.2013

Highlights of Logic, Games and Automata

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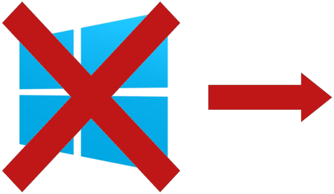
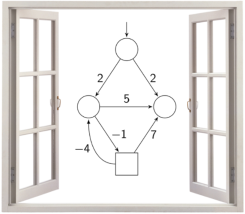
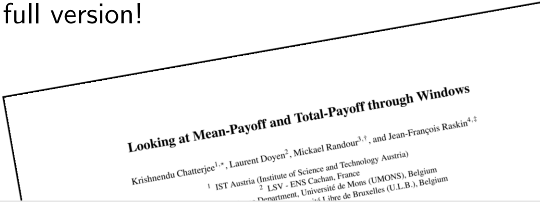
Aim of this talk





1. New family of quantitative objectives, based on MP and TP
2. Convince you of its *advantages* and *usefulness*
3. No technical stuff but feel free to check the conference version (ATVA'13) or the arXiv full version!

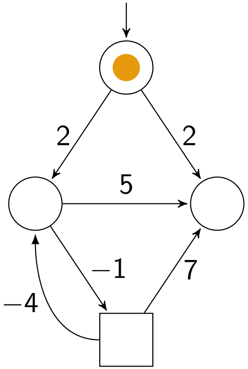


Looking at Mean-Payoff and Total-Payoff through Windows

Krishnendu Chatterjee[1,*], Laurent Doyen[2], Mickael Randour[3,†], and Jean-François Raskin[4,‡]

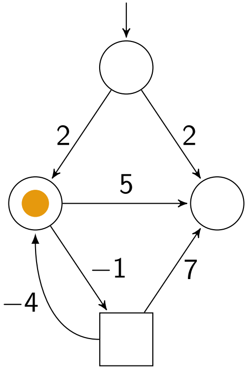
[1] IST Austria (Institute of Science and Technology Austria)
[2] LSV - ENS Cachan, France
Université de Mons (UMONS), Belgium
Libre de Bruxelles (U.L.B.), Belgium

Classical MP and TP games



- $\text{TP}(\pi) = \liminf_{n\to\infty} \sum_{i=0}^{i=n-1} w(s_i, s_{i+1})$
- $\text{MP}(\pi) = \liminf_{n\to\infty} \frac{1}{n} \text{TP}(\pi(n))$

Classical MP and TP games



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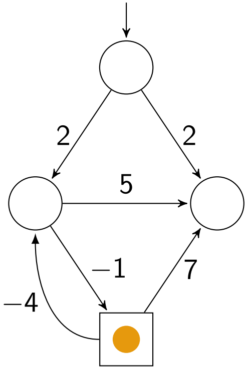

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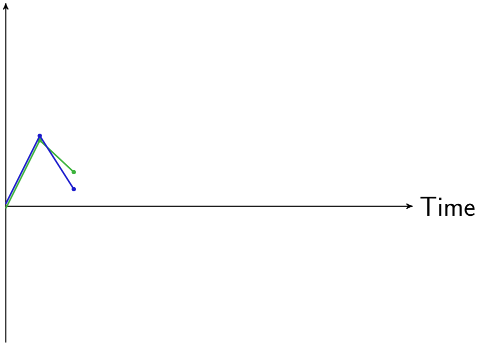

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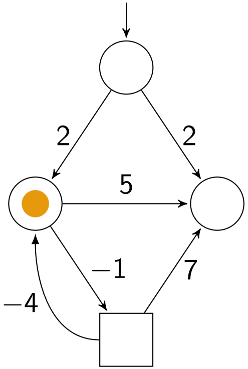

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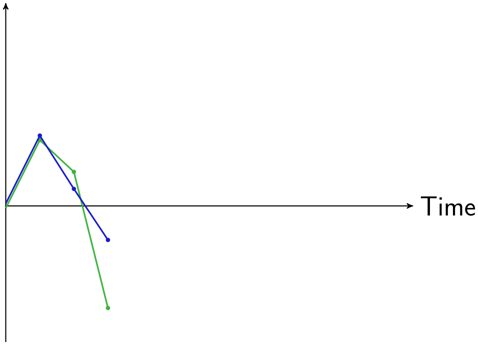

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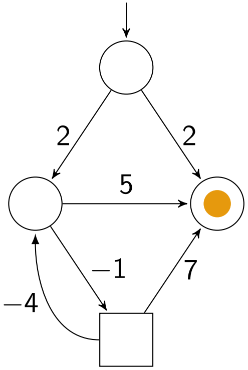

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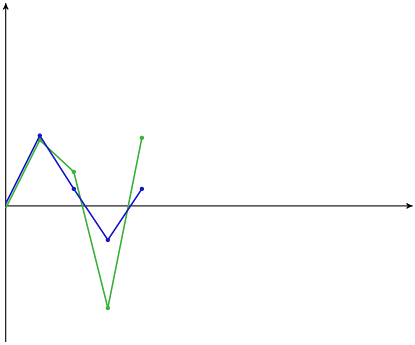

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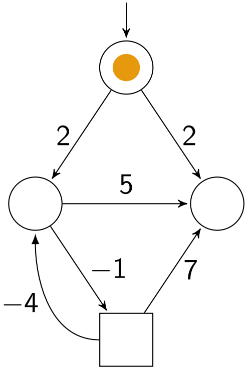

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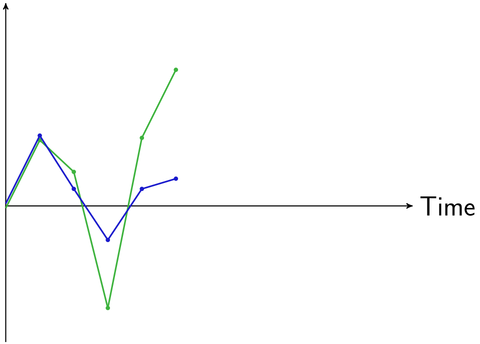

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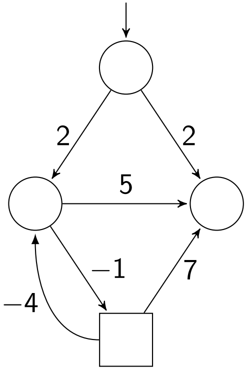

Then, $(2, 5, 2)^\omega$

- $\underline{\text{TP}}(\pi) = \liminf_{n\to\infty} \sum_{i=0}^{i=n-1} w(s_i, s_{i+1})$
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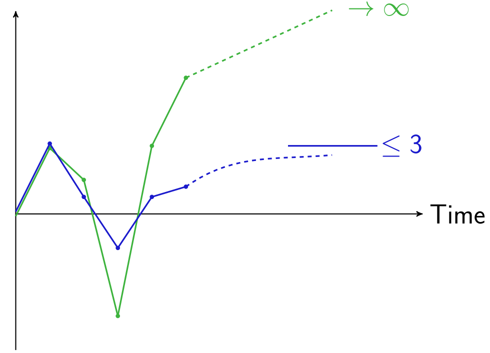

What do we know?

	one-dimension			k-dimension		
	complexity	$\mathcal{P}_1$ mem.	$\mathcal{P}_2$ mem.	complexity	$\mathcal{P}_1$ mem.	$\mathcal{P}_2$ mem.
$\underline{\text{MP}}$ / $\overline{\text{MP}}$	NP $\cap$ coNP	mem-less		coNP-c. / NP $\cap$ coNP	infinite	mem-less
$\underline{\text{TP}}$ / $\overline{\text{TP}}$	NP $\cap$ coNP	mem-less		??	??	??

▷ Long tradition of study. Non-exhaustive selection: [EM79, ZP96, Jur98, GZ04, GS09, CDHR10, VR11, CRR12]

What about multi total-payoff?

	one-dimension			k-dimension		
	complexity	$\mathcal{P}_1$ mem.	$\mathcal{P}_2$ mem.	complexity	$\mathcal{P}_1$ mem.	$\mathcal{P}_2$ mem.
$\underline{\mathsf{MP}}$ / $\overline{\mathsf{MP}}$	NP $\cap$ coNP	mem-less		coNP-c. / NP $\cap$ coNP	infinite	mem-less
$\underline{\mathsf{TP}}$ / $\overline{\mathsf{TP}}$	NP $\cap$ coNP	mem-less		??	??	??

- ▷ TP and MP look very similar in one-dimension
  - TP $\sim$ refinement of MP $= 0$
- ▷ Is it still true in multi-dimension?

What about multi total-payoff?

	one-dimension			k-dimension		
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▷ Unfortunately, no!

It would be nice to have...

a **decidable** objective with the same flavor (some sort of approx.)

Is the complexity barrier breakable?

	one-dimension			k-dimension		
	complexity	$\mathcal{P}_1$ mem.	$\mathcal{P}_2$ mem.	complexity	$\mathcal{P}_1$ mem.	$\mathcal{P}_2$ mem.
$\underline{\text{MP}}$ / $\overline{\text{MP}}$	NP ∩ coNP	mem-less		coNP-c. / NP ∩ coNP	infinite	mem-less
$\underline{\text{TP}}$ / $\overline{\text{TP}}$	NP ∩ coNP	mem-less		**Undec.**	-	-

▷ P membership for the one-dim. case is a long-standing open problem!

It would be nice to have...

an approximation decidable in **polynomial time**

Do we *really* want to play eternally?

	one-dimension			k-dimension		
	complexity	$\mathcal{P}_1$ mem.	$\mathcal{P}_2$ mem.	complexity	$\mathcal{P}_1$ mem.	$\mathcal{P}_2$ mem.
$\underline{\text{MP}}$ / $\overline{\text{MP}}$	$\text{NP} \cap \text{coNP}$	mem-less		coNP-c. / $\text{NP} \cap \text{coNP}$	infinite	mem-less
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- ▷ MP and TP give *no timing guarantee*: the "good behavior" occurs at the limit...
- ▷ Sure, in one-dim., memoryless strategies suffice and provide bounds on cycles, but what if we are given an arbitrary play?

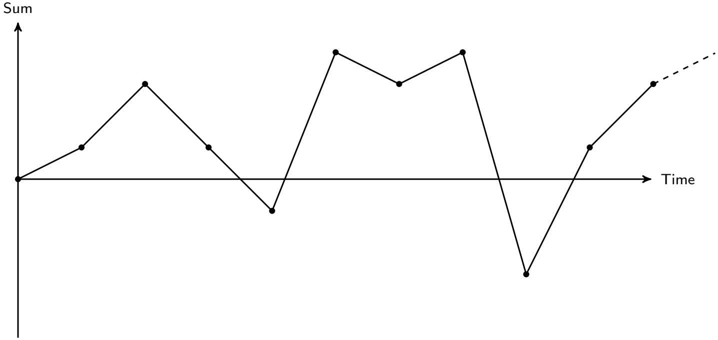
It would be nice to have...

a quantitative measure that **specifies timing requirements**

Window objectives: key idea

- **Window** of fixed size **sliding** along a play
  $\rightsquigarrow$ defines a local finite horizon
- Objective: see a **local** $MP \geq 0$ *before hitting the end* of the window
  $\rightsquigarrow$ needs to be verified at *every* step

Window MP, threshold zero, maximal window = 4



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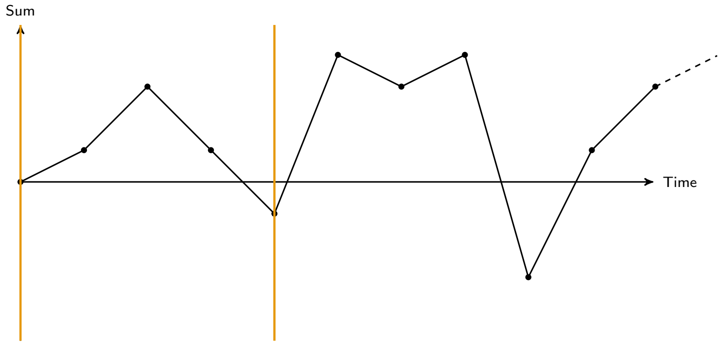

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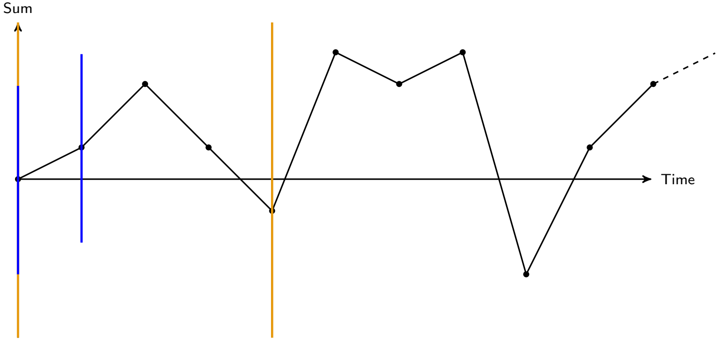

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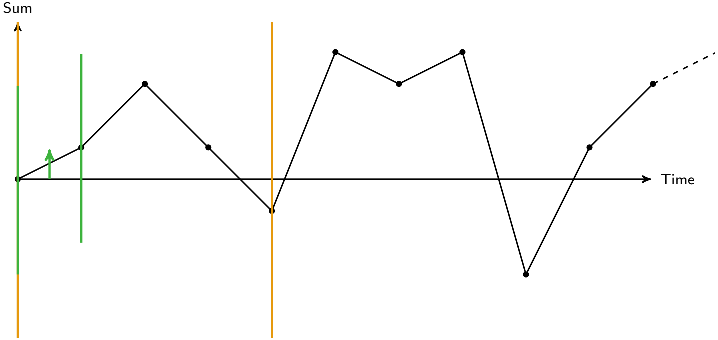

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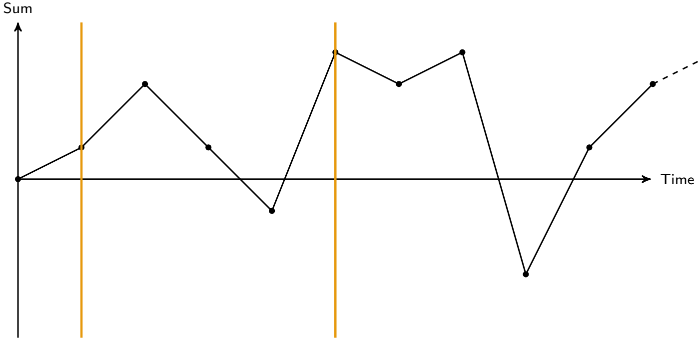

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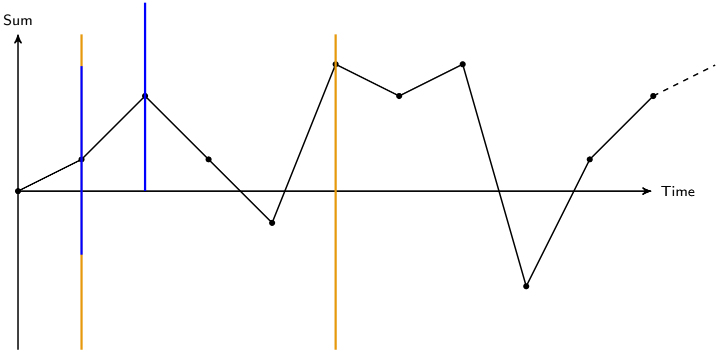

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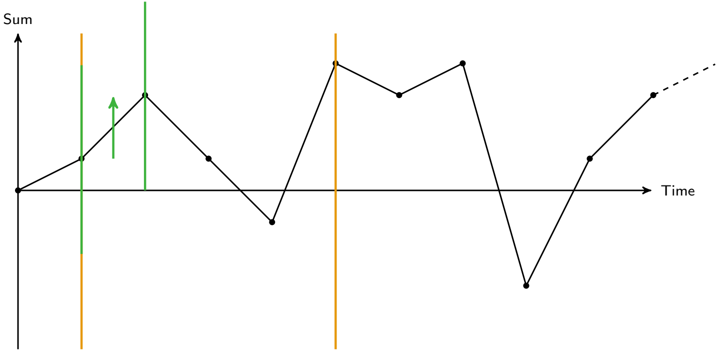

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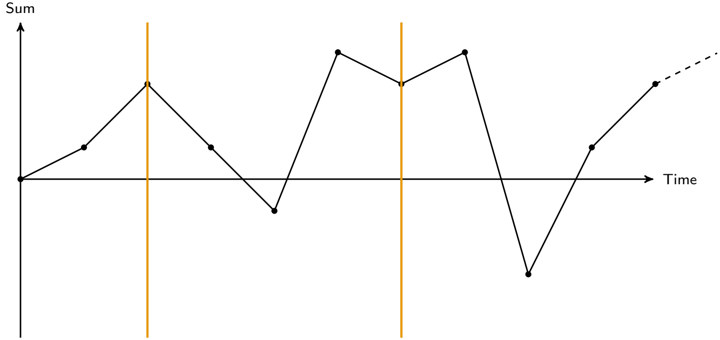

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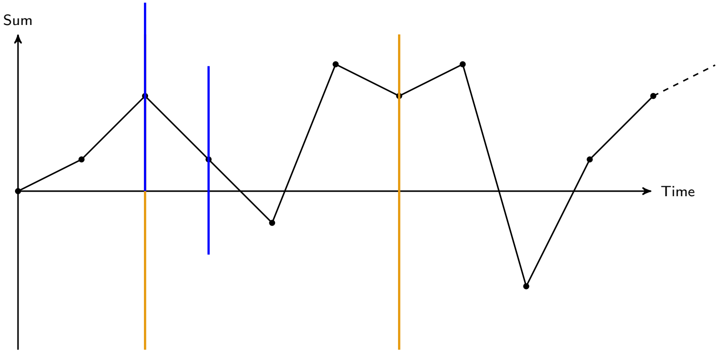

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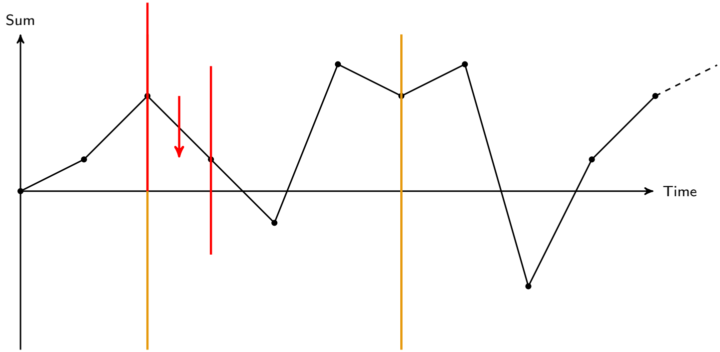

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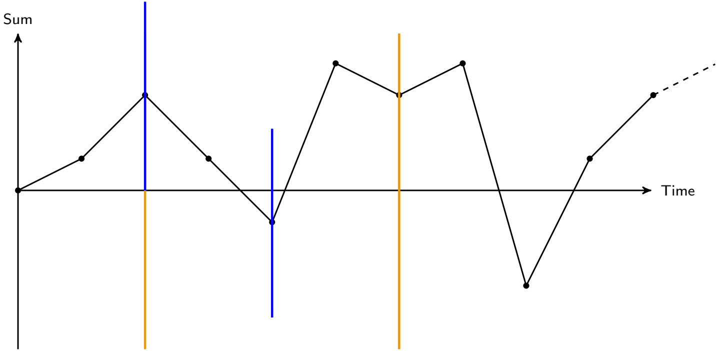

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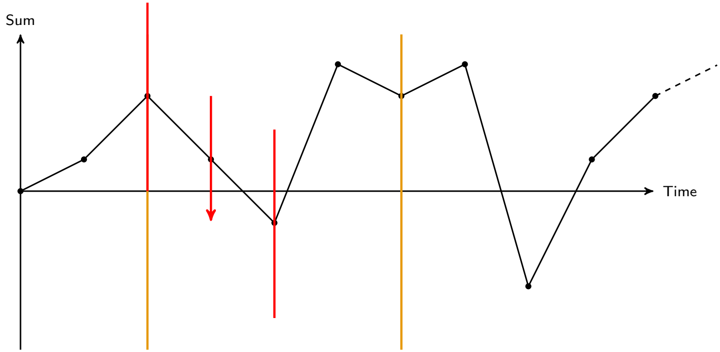

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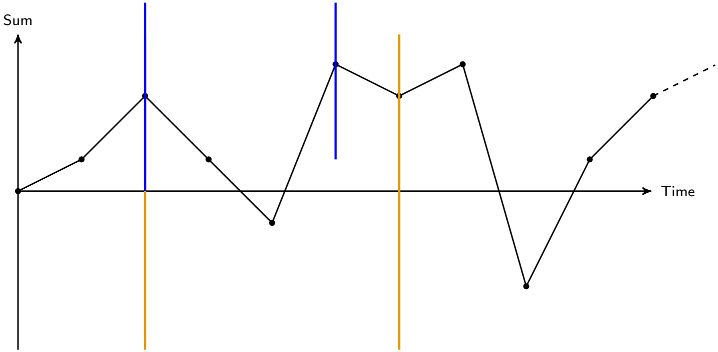

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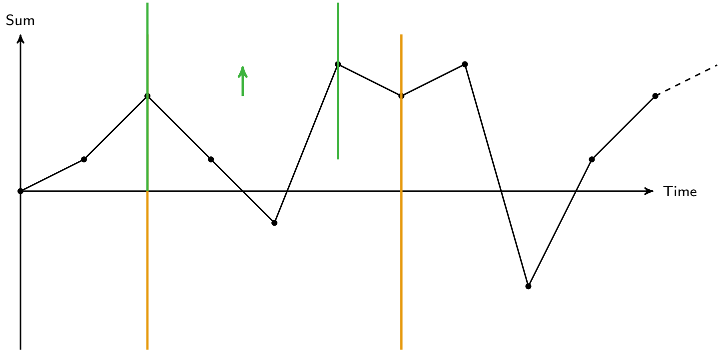

Multiple variants

- Given $l_{\max} \in \mathbb{N}_0$, *good window* $\mathbf{GW}(l_{\max})$ asks for a positive sum in at most $l_{\max}$ steps (one window, from the first state)
- *Direct Fixed Window*: $\mathbf{DFW}(l_{\max}) \equiv \Box\mathbf{GW}(l_{\max})$
- *Fixed Window*: $\mathbf{FW}(l_{\max}) \equiv \Diamond\mathbf{DFW}(l_{\max})$
- *Direct Bounded Window*: $\mathbf{DBW} \equiv \exists\, l_{\max},\ \mathbf{DFW}(l_{\max})$
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Conservative approximations in one-dim.

$$\text{Any window obj.} \Rightarrow \mathbf{BW} \Rightarrow \text{MP} \geq 0$$
$$\mathbf{BW} \Leftarrow \text{MP} > 0$$

Results overview

<table>
<tr><th></th><th colspan="3">one-dimension</th><th colspan="3">k-dimension</th></tr>
<tr><th></th><th>complexity</th><th>$\mathcal{P}_1$ mem.</th><th>$\mathcal{P}_2$ mem.</th><th>complexity</th><th>$\mathcal{P}_1$ mem.</th><th>$\mathcal{P}_2$ mem.</th></tr>
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<tr><td>WMP: fixed polynomial window</td><td>P-c.</td><td colspan="2" rowspan="2">mem. req.
≤ linear($|S| \cdot l_{\max}$)</td><td>PSPACE-h.
EXP-easy</td><td colspan="2" rowspan="2">exponential</td></tr>
<tr><td>WMP: fixed arbitrary window</td><td>P($|S|, V, l_{\max}$)</td><td>EXP-c.</td></tr>
<tr><td>WMP: bounded window problem</td><td>NP ∩ coNP</td><td>mem-less</td><td>infinite</td><td>NPR-h.</td><td>-</td><td>-</td></tr>
</table>

▷ $|S|$ the # of states, V the length of the binary encoding of weights, and $l_{\max}$ the window size

Results overview: advantages

	one-dimension			k-dimension		
	complexity	$\mathcal{P}_1$ mem.	$\mathcal{P}_2$ mem.	complexity	$\mathcal{P}_1$ mem.	$\mathcal{P}_2$ mem.
$\underline{\text{MP}}$ / $\overline{\text{MP}}$	NP $\cap$ coNP	mem-less		coNP-c. / NP $\cap$ coNP	infinite	mem-less
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WMP: fixed polynomial window	**P-c.**	**mem. req.** $\leq$ **linear**$(\lvert S\rvert \cdot l_{\max})$		**PSPACE-h.** **EXP-easy**	**exponential**	
WMP: fixed arbitrary window	**P**$(\lvert S\rvert, V, l_{\max})$			**EXP-c.**		
WMP: bounded window problem	**NP $\cap$ coNP**	**mem-less**	**infinite**	**NPR-h.**	-	-

- ▷ $|S|$ the $\#$ of states, V the length of the binary encoding of weights, and $l_{\max}$ the window size
- ▷ For one-dim. games with poly. windows, we are in **P**
- ▷ For multi-dim. games with fixed windows, we are **decidable**
- ▷ Window obj. provide **timing guarantees**

Taste of the proofs ingredients

- For those who like it technical, we use
  - ▷ 2CMs [Min61],
  - ▷ membership problem for APTMs [CKS81],
  - ▷ countdown games [JSL08] ,
  - ▷ generalized reachability [FH10],
  - ▷ reset nets [DFS98, Sch02, LNO+08],
  - ▷ ...
- *Open question*: is bounded window decidable in multi-dim. ?

Thanks to the Highlights Team

Check the full version on arXiv! abs/1302.4248

Thanks!

Do not hesitate to discuss with us!

K. Chatterjee, L. Doyen, T.A. Henzinger, and J.-F. Raskin.
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