Extending finite-memory determinacy by Boolean combination of winning conditions

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The talk in one slide

Strategy synthesis for two-player turn-based games

Finding **good** controllers for systems interacting with an *antagonistic* environment.

- Good? Performance evaluated through *objectives* / *payoffs*.

Question

When are *simple* strategies sufficient to play optimally?

- We establish a general framework that preserves **finite-memory determinacy** when combining objectives.
- Joint work with S. Le Roux and A. Pauly, in FSTTCS'18 [RPR18] (on arXiv).

1 Memoryless determinacy

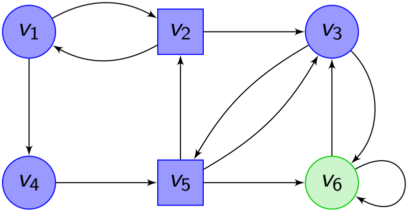
2 Finite-memory determinacy and Boolean combinations

3 Conclusion and ongoing work

Games on graphs: example

We consider *finite* arenas with vertex *colors* in C. Two players: circle (1) and square (2). Strategies $C^* \times V_i \to V$ (w.l.o.g.).

- A **winning condition** is a set $W \subseteq C^\omega$.



From where can Player 1 ensure to reach v_6? How complex is his strategy?

Memoryless strategies ($V_i \to V$) always suffice for reachability (for both players).

When are memoryless strategies sufficient to play optimally?

Virtually always for **simple** winning conditions!

Examples: reachability, safety, Büchi, parity, mean-payoff, energy, total-payoff, average-energy, etc.

Can we characterize when they are?

Yes, thanks to Gimbert and Zielonka [GZ05] (see also, e.g., [Kop06, AR17]).

Gimbert and Zielonka's criterion

Memoryless strategies suffice for a *preference relation* (and the induced winning conditions) iff

1. it is **monotone**,
   - ▷ Intuitively, stable under prefix addition.
2. it is **selective**.
   - ▷ Intuitively (the true characterization is slightly more subtle), stable under cycle mixing.

Example: reachability.

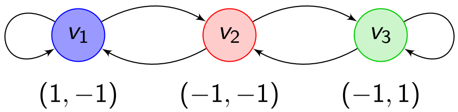
No equivalent for finite memory!

I will come back to that. . . ☺

Combining winning conditions (1/2)

Needed for multi-objective reasoning.

Memoryless strategies do not suffice anymore, even for simple conjunctions!



Examples:

- Büchi for v_1 *and* v_3 $\rightarrow$ **finite** (1 bit) memory.
- Mean-payoff (average weight per transition) ≥ 0 on all dimensions $\rightarrow$ **infinite** memory!

Combining winning conditions (2/2)

Our goal

We want a *general* and *abstract* theorem guaranteeing the sufficiency of finite-memory strategies[a] in games with Boolean combinations of objectives provided that the underlying simple objectives fulfill some criteria.

[a] Implementable via a finite-state machine.

Combining winning conditions (2/2)

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Advantages:

- study of core features ensuring finite-memory determinacy,
- works for almost all existing settings and many more to come.

Drawbacks:

- concrete memory bounds are huge (as they depend on the most general upper bound).
- sufficient criterion, not full characterization.

The building blocks

The full approach is technically involved but can be sketched intuitively.

Criterion outline

Any *well-behaved* winning condition combined with conditions traceable by finite-state machines (i.e., *safety-like* conditions) preserves finite-memory determinacy.

To state this theorem formally, we need three ingredients:

1. *regularly-predictable* winning conditions,
2. *regular* languages,
3. *hypothetical* subgame-perfect equilibria (hSPE).

We match the FM-determinacy frontier almost exactly!

$\Longrightarrow$ **Only one exception AFAWK (hSPE vs. opt. strategies).**

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Conclusion

- Combining similar simple objectives leads to contrasting behaviors: difficult to extract the core features leading to FM determinacy.
- Our main result is a **sufficient criterion**, not a full characterization.
  - ▷ In practice, it does cover everything except *average-energy with a lower-bounded energy condition* – a very strange corner case.
  - ▷ **Any weakening of our hypotheses almost immediately leads to falsification.**
  - ▷ We also have several **more precise results** (e.g., much lower bounds) for specific combinations and/or restrictive hypotheses.

Ongoing work

We now have an almost complete picture of the frontiers of FM determinacy for *combinations of objectives*.

What about a complete characterization à la Gimbert and Zielonka?

Ongoing work with P. Bouyer, S. Le Roux, Y. Oualhadj and P. Vandenhove. Promising preliminary results.

Thank you! Any question?

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